

2006/09

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Nature



Danmark

350

$$9) \sin^2\left(\frac{\pi}{4} + \frac{x}{2}\right) - 2\sin^2\left(\frac{3}{4}\pi + \frac{x}{2}\right) = \sin\frac{7}{6}\pi \sin 3x$$

$$\frac{1 - \cos\left(\frac{\pi}{2} + x\right)}{2} - \frac{1 - \cos\left(\frac{3}{2}\pi + x\right)}{2} = -\frac{1}{2} \sin 3x$$

$$1 + \sin x - 1 + \sin x = \sin 3x \Rightarrow \sin 3x = 0$$

$$\sin x + 2\sin 2x \cos x = 0$$

$$\sin x (1 + 4\cos^2 x) = 0$$

$$\left\{ \begin{array}{l} \sin x = 0 \\ 1 + 4\cos^2 x = 0 \end{array} \right. \quad x = \pi k; k \in \mathbb{Z}$$

$$x \in \emptyset$$

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$$u) \cos \alpha = \frac{12}{13}$$

$$u \quad \frac{3}{2\pi} < \alpha < 2\pi \Rightarrow \alpha \in \text{IV quadrant.}$$

$$\frac{1 + \operatorname{tg} \alpha}{1 + \operatorname{ctg} \alpha} = \frac{1 + \operatorname{tg} \alpha}{1 + \frac{1}{\operatorname{tg} \alpha}} = \frac{1 + \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha} \cdot \operatorname{tg} \alpha = \operatorname{tg} \alpha$$

$$\operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha} - 1 = \frac{169}{144} - 1 = \frac{25}{144} = \sqrt{\frac{5}{12}}$$

$$\operatorname{tg} \alpha = -\frac{5}{12}$$

$$\frac{1 + \operatorname{tg} \alpha}{1 + \operatorname{ctg} \alpha} = \operatorname{tg} \alpha = -\frac{5}{12}$$

$$p) \sin \alpha = \frac{4}{5} \quad u \quad \frac{\pi}{2} < \alpha < \pi \Rightarrow \alpha \in \text{II quadrant}$$

$$\frac{2 - 3 \operatorname{tg} \alpha}{3 - 2 \operatorname{ctg} \alpha} = \left(2 - \frac{3}{\operatorname{ctg} \alpha} \right) : \frac{3 - 2 \operatorname{ctg} \alpha}{\operatorname{ctg} \alpha} = \frac{2 \operatorname{ctg} \alpha - 3}{\operatorname{ctg} \alpha} \cdot \left(\frac{1}{3 \operatorname{ctg} \alpha - 3} \right) =$$

$$= -\frac{1}{\operatorname{ctg} \alpha} \cdot \frac{2 - \operatorname{tg} \alpha}{1} =$$

$$\operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha} - 1 = \frac{25}{16} - 1 = \frac{9}{16}$$

$$\operatorname{ctg} \alpha = -\frac{3}{4} \quad \vee \quad \operatorname{tg} \alpha = \frac{4}{3}$$

$$\frac{2 - 3 \operatorname{tg} \alpha}{3 - 2 \operatorname{ctg} \alpha} = \frac{2 - \operatorname{tg} \alpha}{1} = \frac{4}{3}$$

$$6) \cos \lambda = -\frac{1}{3} \quad \text{и} \quad \pi < \lambda < \frac{3\pi}{2} \quad \text{и} \quad \lambda \in \text{III} \text{ квад.}$$

$$\frac{\cos \lambda + \operatorname{ctg} \lambda}{\operatorname{ctg} \lambda}, \quad \frac{\cos \lambda}{\operatorname{ctg} \lambda} + 1 = \sin \lambda + 1$$

$$\sin^2 \lambda, \quad 1 - \cos^2 \lambda = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\sin \lambda = -\frac{\sqrt{2}}{3}$$

$$\frac{\cos \lambda + \operatorname{ctg} \lambda}{\operatorname{ctg} \lambda} = 1 + \sin \lambda = 1 - \frac{\sqrt{2}}{3}, \quad \frac{3 - \sqrt{2}}{3}$$

$$7) \sin \lambda = -\frac{1}{3} \quad \text{и} \quad \pi < \lambda < \frac{3\pi}{2} \quad \text{и} \quad \lambda \in \text{III} \text{ квад.}$$

$$\frac{\sin \lambda + \operatorname{tg} \lambda}{\operatorname{tg} \lambda}, \quad \frac{\sin \lambda}{\operatorname{tg} \lambda} + 1 = \cos \lambda + 1$$

$$\cos^2 \lambda = 1 - \sin^2 \lambda = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\cos \lambda = -\frac{\sqrt{2}}{3}$$

$$\frac{\sin \lambda + \operatorname{tg} \lambda}{\operatorname{tg} \lambda} = \cos \lambda + 1 = 1 - \frac{\sqrt{2}}{3}, \quad \frac{3 - \sqrt{2}}{3}$$

2/урд 59

$$\text{uf } \sin \lambda = \frac{\sqrt{65}}{9}$$

$$\text{u } \begin{cases} \operatorname{tg} \lambda > 0 \\ \sin > 0 \end{cases}$$

$\Rightarrow \lambda \in I$ дун.

$$\cos^2 \lambda = 1 - \sin^2 \lambda = 1 - \frac{65}{81} = \frac{16}{81}$$

$$\cos \lambda = \frac{4}{9}$$

$$\text{Дун! } \cos \lambda = \frac{4}{9}$$

$$f) \operatorname{ctg} \lambda = -2\sqrt{2}$$

$$\text{u } \begin{cases} \cos \lambda < 0 \\ \operatorname{ctg} \lambda < 0 \end{cases}$$

$\Rightarrow \lambda \in II$ дун

$$\text{Антет. } 9 \sin \lambda = 2$$

$$\sin^2 \lambda = \frac{1}{1 + \operatorname{ctg}^2 \lambda} = \frac{1}{1 + 8} = \frac{1}{9}$$

$$\sin \lambda = \frac{1}{3} \Rightarrow 9 \sin \lambda = 3$$

$$\text{Дун! } 9 \sin \lambda = 3$$

$$g) \operatorname{tg} \lambda = 2\sqrt{2}$$

$$\text{u } \begin{cases} \sin \lambda > 0 \\ \operatorname{tg} \lambda > 0 \end{cases}$$

$\Rightarrow \lambda \in I$ дун.

$$\text{Антет. } 3 \cos \lambda$$

$$\cos^2 \lambda = \frac{1}{1 + \operatorname{tg}^2 \lambda} = \frac{1}{9}$$

$$\cos \lambda = \frac{1}{3} \Rightarrow 3 \cos \lambda = 1$$

$$\text{Дун! } 3 \cos \lambda = 1$$

2/урнд 60

$$\operatorname{tg} x + \operatorname{ctg} x = 3$$

$$\text{w/ } \operatorname{tg}^2 x + \operatorname{ctg}^2 x = (\operatorname{tg} x + \operatorname{ctg} x)^2 - 2 \operatorname{tg} x \operatorname{ctg} x = 9 - 2 = 7$$

$$A) \operatorname{tg}^3 x + \operatorname{ctg}^3 x = (\operatorname{tg} x + \operatorname{ctg} x)(\operatorname{tg}^2 x - \operatorname{tg} x \operatorname{ctg} x + \operatorname{ctg}^2 x) =$$

$$= (\operatorname{ctg} x + \operatorname{tg} x)((\operatorname{tg} x + \operatorname{ctg} x)^2 - 3 \operatorname{tg} x \operatorname{ctg} x) =$$

$$= 3(9 - 3) = 18$$

$$B) \operatorname{tg}^4 x + \operatorname{ctg}^4 x = (\operatorname{tg}^2 x + \operatorname{ctg}^2 x)^2 - 2 \operatorname{tg}^2 x \cdot \operatorname{ctg}^2 x =$$

$$= ((\operatorname{tg} x + \operatorname{ctg} x)^2 - 2 \operatorname{tg} x \operatorname{ctg} x)^2 - 2 \operatorname{tg}^2 x \cdot \operatorname{ctg}^2 x =$$

$$= 9 - 4 = 5 - \cancel{48} \quad 4^4$$

2/урнд 61

$$\sin x + \cos x = 0,8$$

$$\text{w/ } \sin x \cdot \cos x$$

$$(\sin x + \cos x)^2 = 0,64$$

$$\sin^2 x + \cos^2 x + 2 \sin x \cos x = 0,64$$

$$\sin x \cdot \cos x = \frac{0,64 - 1}{2} = -0,18$$

$$P) \sin^3 x + \cos^3 x = (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x) =$$

$$= 0,8(1 + 0,18) = 0,8 \cdot 1,18 = 0,944$$

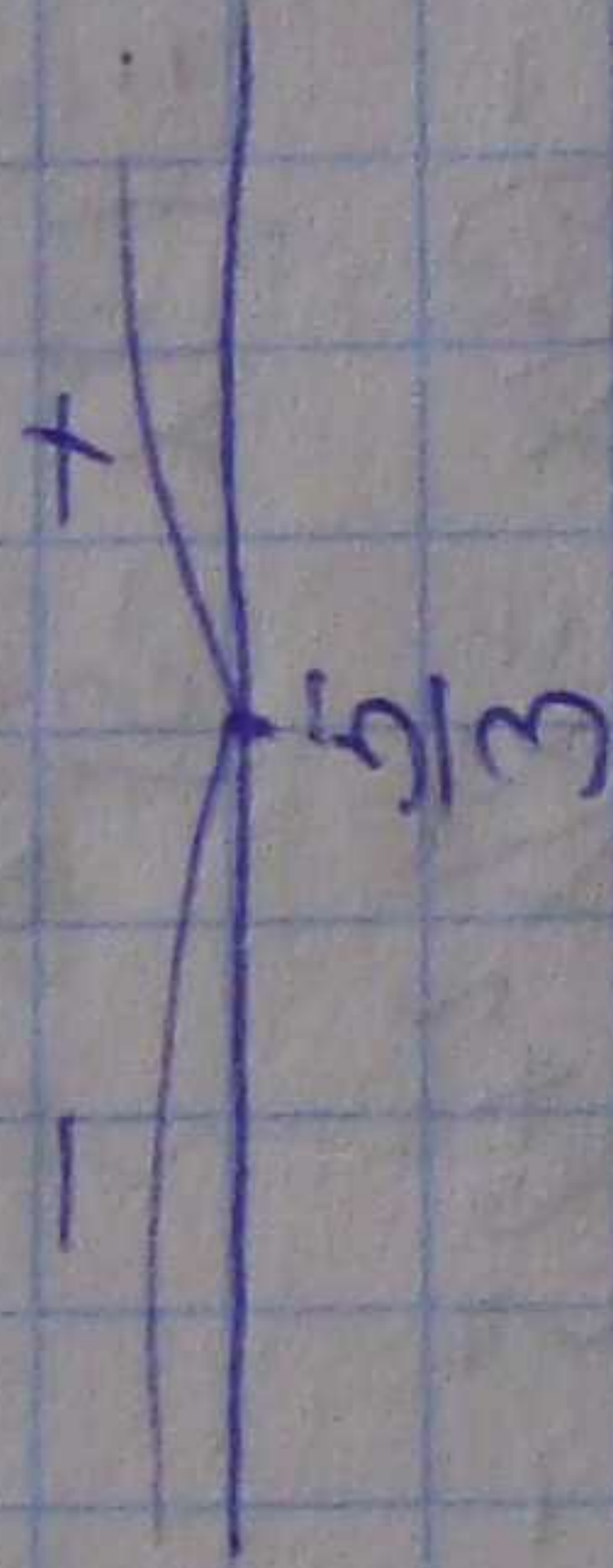
$$\begin{aligned}
 g) \sin^4 x + \cos^4 x &= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x = \\
 &= ((\sin x + \cos x)^2 - 2\sin x \cos x)^2 - 2(\sin x \cos x)^2 = \\
 &= (0,8^2 + 2 \cdot 0,18)^2 - 2 \cdot 0,18^2 = (0,64 + 0,36)^2 - \\
 &- 0,0648 = 1 - 0,0648 = 0,9352
 \end{aligned}$$

Задача 63

$$u) |3x - 5| = 7$$

$$3x - 5 = 0$$

$$x = \frac{5}{3}$$



$$\begin{cases} x \geq \frac{5}{3} \\ 3x - 5 = 7 \end{cases} \Rightarrow \begin{cases} x \in \left[\frac{5}{3}; +\infty\right) \\ x = 4 \end{cases} \Rightarrow x = 4$$

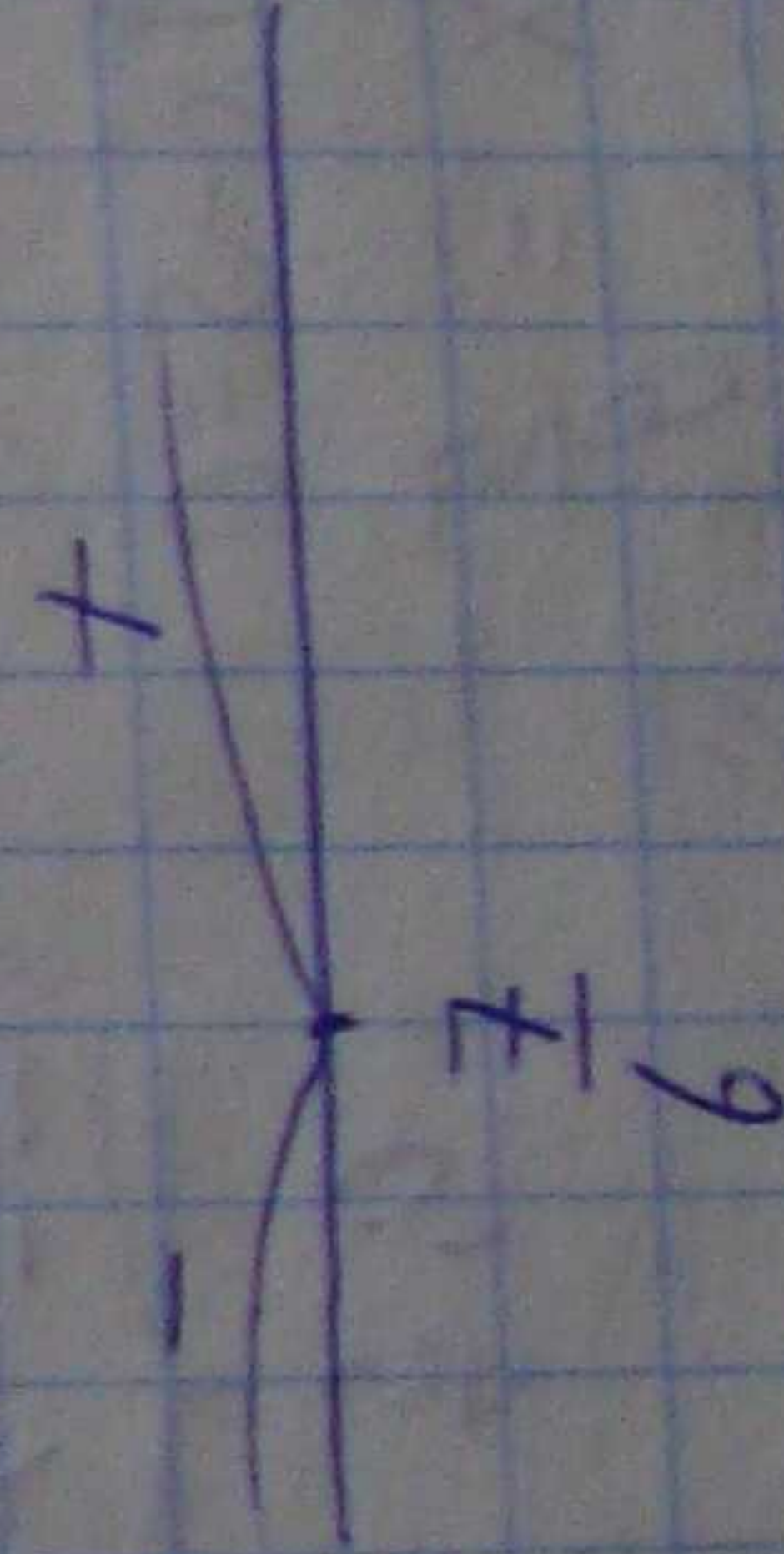
$$\begin{cases} x < \frac{5}{3} \\ 5 - 3x = 7 \end{cases} \Rightarrow \begin{cases} x \in (-\infty; \frac{5}{3}) \\ x = -\frac{2}{3} \end{cases} \Rightarrow x = -\frac{2}{3}$$

Ответ: $x = 4; -\frac{2}{3}$

$$F) |6x - 7| = 7x + 1$$

$$6x - 7 = 0$$

$$x = \frac{7}{6}$$



$$\left\{ \begin{array}{l} x \geq \frac{7}{6} \\ 6x - 7 = 7x + 1 \\ x < \frac{7}{6} \\ 7 - 6x = 7x + 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} x \in [\frac{7}{6}, +\infty) \\ x = -8 \\ x \in (-\infty, \frac{7}{6}) \\ x = \frac{6}{13} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} x \in \emptyset \\ x = \frac{6}{13} \end{array} \right\}$$

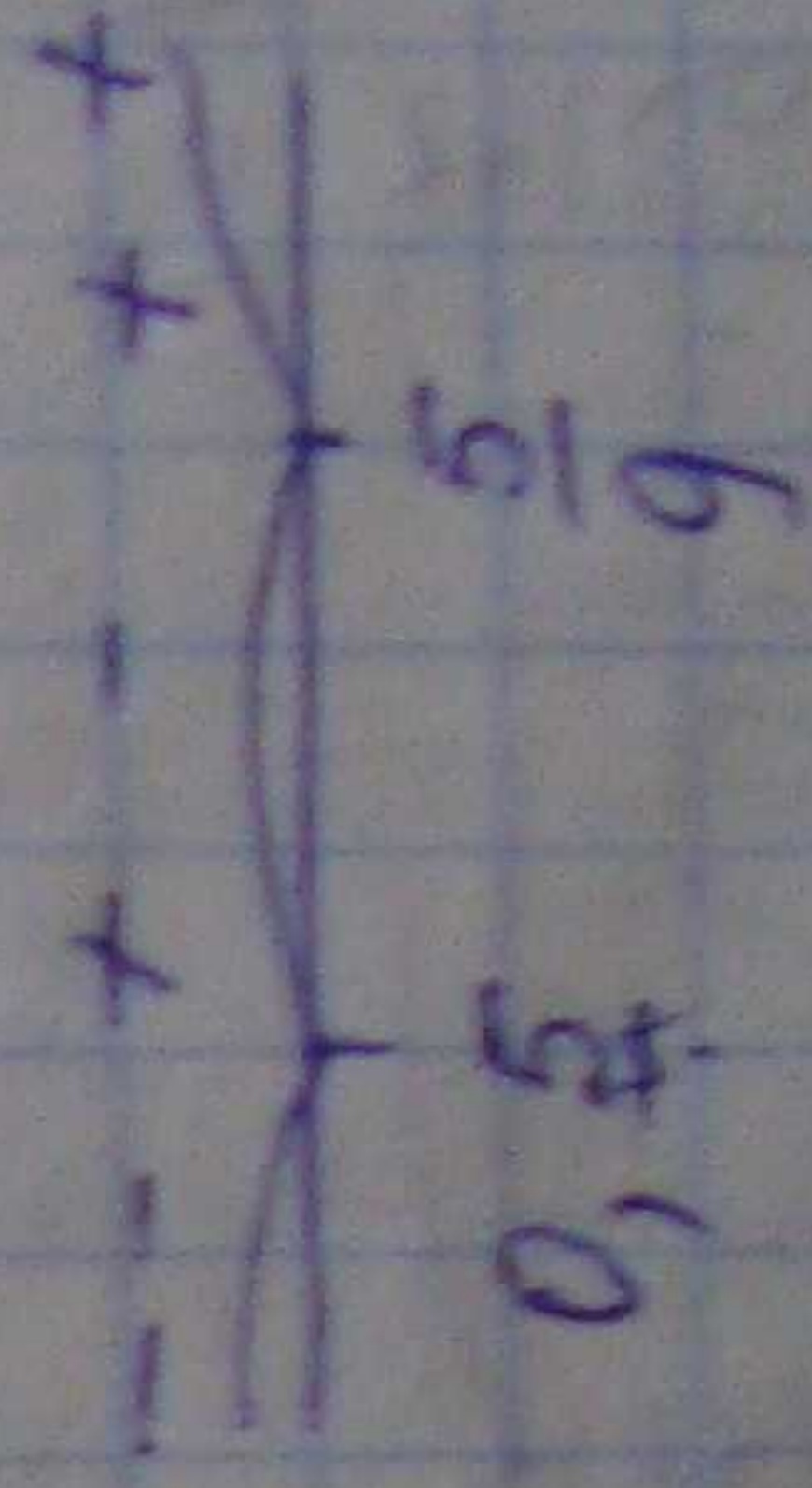
$$\text{min: } x = \frac{6}{13}$$

$$\begin{aligned} 4) |8x-4| &= |9x-5| \\ (8x-4)^2 &= (9x-5)^2 \\ 64x^2 + 16 - 64x &= 81x^2 + 25 - 90x \\ 4x^2 - 4x + 1 &= 81x^2 - 90x + 25 \\ 73x^2 - 86x + 24 &= 0 \end{aligned}$$

$$\Delta = 4^2 - 4 \cdot 73 = 97$$

$$x_{1,2} = \frac{24 \pm \sqrt{97}}{73}$$

$$\begin{aligned} |8x-4| &= |9x-5| \\ |8x-4| - |9x-5| &= 0 \\ 8x-4 &= 0 & 9x-5 &= 0 \\ x &= 0,5 & x &= \frac{5}{9} \end{aligned}$$



$$\begin{cases}
 x < 0,5 \\
 4 - 8x + 9x - 5 = 0 \\
 9,5 \leq x \leq \frac{5}{9} \\
 8x - 4 + 9x - 5 = 0 \\
 x > \frac{5}{9} \\
 8x - 4 - 9x + 5 = 0
 \end{cases}
 \Rightarrow
 \begin{cases}
 x \in (-\infty, 0,5) \\
 x = 1 \\
 x \in [0,5, \frac{5}{9}) \\
 x = \frac{9}{14} \\
 x \in (\frac{5}{9}, +\infty) \\
 x = 1
 \end{cases}
 \Rightarrow
 \begin{cases}
 x \in \emptyset \\
 x = \frac{9}{14} \\
 x = 1
 \end{cases}$$

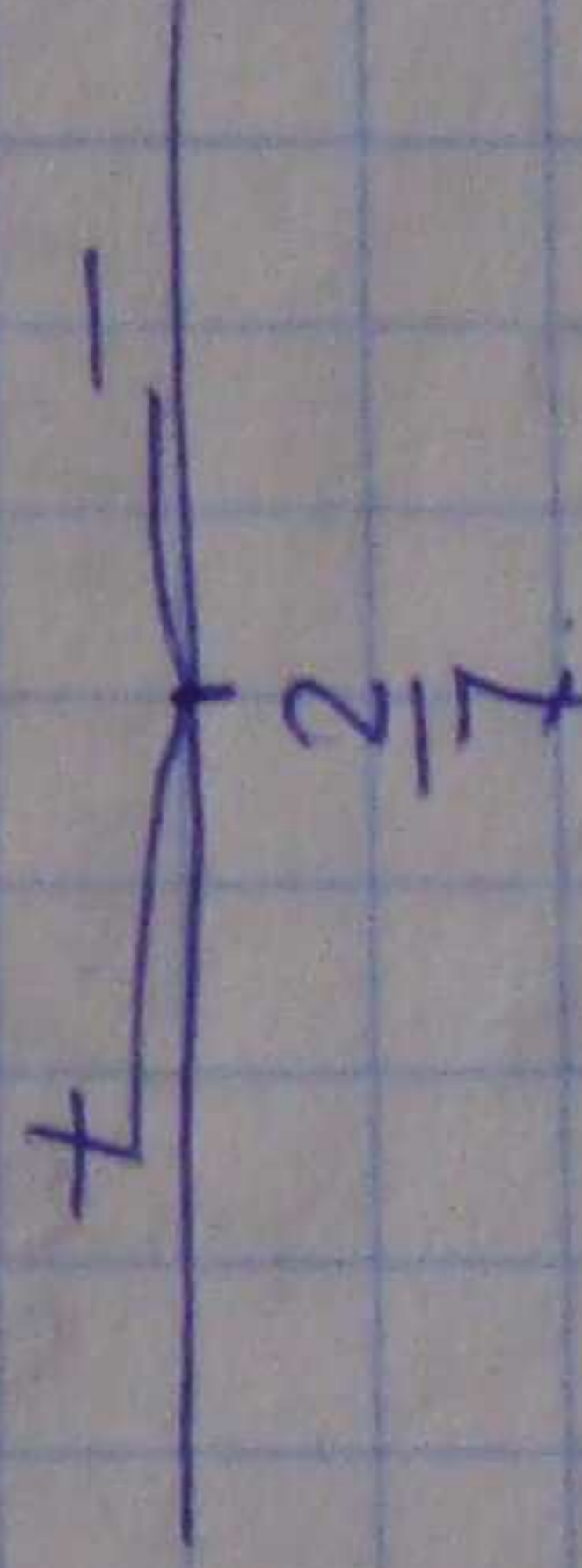
$$\text{Für } x = \frac{9}{14}; 1$$

Summe 64

$$\text{iii) } |2 - 7x| < 5$$

$$2 - 7x = 0$$

$$x = \frac{2}{7}$$



$$\begin{cases}
 x \geq \frac{2}{7} \\
 7x - 2 < 5
 \end{cases}
 \Rightarrow
 \begin{cases}
 x \in [\frac{2}{7}, +\infty) \\
 x \in (-\infty, 1)
 \end{cases}
 \Rightarrow x \in [\frac{2}{7}, 1)$$

$$\begin{cases}
 x < \frac{2}{7} \\
 2 - 7x < 5
 \end{cases}
 \Rightarrow
 \begin{cases}
 x \in (-\infty, \frac{2}{7}) \\
 x \in (-\frac{3}{7}, +\infty)
 \end{cases}
 \Rightarrow x \in (-\frac{3}{7}, \frac{2}{7})$$

$$\Rightarrow x \in (-\frac{3}{7}, 1)$$

$$\text{Für } x \in (-\frac{3}{7}, 1)$$

$$f) |5x - 5| \geq 2$$

$$5x - 5 = 0$$

$$x = 1$$



$$\left\{ \begin{array}{l} 5x \cdot x \leq 1 \\ 5 - 5x \geq 2 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} x \in (-\infty; 1] \\ x \in (-\infty; 0,8] \end{array} \right\} \Rightarrow x \in (-\infty; 0,8]$$

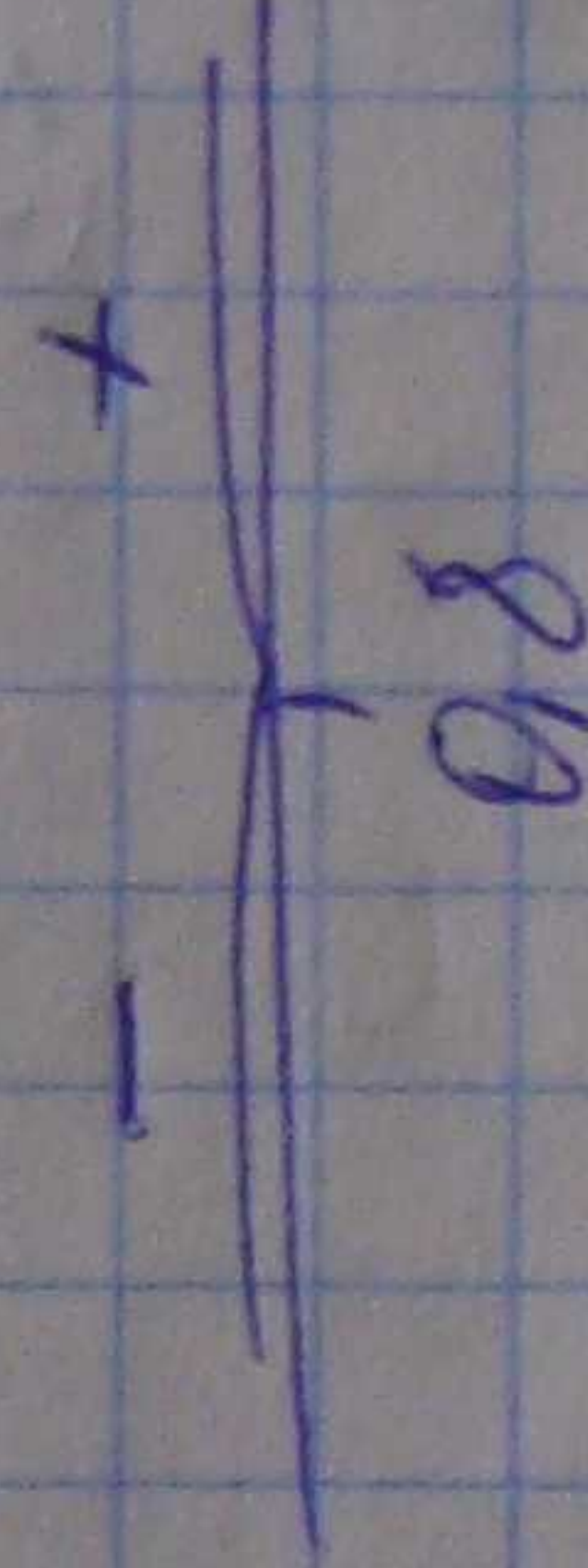
$$\left\{ \begin{array}{l} x > 1 \\ 5x - 5 \geq 2 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} x \in (1; +\infty) \\ x \in [1,4; +\infty) \end{array} \right\} \Rightarrow x \in [1,4; +\infty)$$

$$x \in (-\infty; 0,8] \cup [1,4; +\infty)$$

$$g) |5x - 4| \leq x$$

$$5x - 4 = 0$$

$$x = 0,8$$



$$\left\{ \begin{array}{l} 5x \leq 0,8 \\ 4 - 5x \leq x \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} x \in (-\infty; 0,8] \\ x \in [\frac{2}{3}; +\infty) \end{array} \right\} \Rightarrow x \in [\frac{2}{3}; 0,8]$$

$$\left\{ \begin{array}{l} x > 0,8 \\ 5x - 4 \leq x \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} x \in (0,8; +\infty) \\ x \in (-\infty; 1] \end{array} \right\} \Rightarrow x \in (0,8; 1]$$

$$x \in [\frac{2}{3}; 1]:$$

2/sep 76

$$2) \cos^2\left(\frac{5\pi}{2} + \alpha\right) - 2\sin\alpha + 1 = 0.$$

$$\cos^2\left(\frac{\pi}{2} + \alpha\right) - 2\sin\alpha + 1 = 0$$

$$\sin^2\alpha - 2\sin\alpha + 1 = 0$$

$$\sin\alpha (\sin\alpha - 1)^2 = 0$$

$$\sin\alpha - 1 = 0.$$

$$\sin\alpha = 1$$

2/sep 85.

$$\text{we} \cos\left(\frac{\pi}{4} - \alpha\right) = \cos\frac{\pi}{4}\cos\alpha + \sin\frac{\pi}{4}\sin\alpha =$$

$$= \frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$$

$$\text{we} \cos\left(\frac{\pi}{4} - \alpha\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$A) \cos\left(\frac{\pi}{4} + \alpha\right) = \cos\frac{\pi}{4}\cos\alpha - \sin\frac{\pi}{4}\sin\alpha = \\ = \frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}(\sin\alpha - \cos\alpha)$$

28.11.2009

19.02.2009

$$p) \sin(\alpha - \beta)$$

$$\sin \alpha = \frac{24}{25}$$

$$\cos \beta = \frac{4}{5}$$

$$0 < \alpha < \frac{\pi}{2}$$

$$\frac{\pi}{2} < \beta < \pi$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{24^2}{25^2}} = \sqrt{\frac{25^2 - 24^2}{25^2}} = \frac{7}{25}$$

$$\frac{7}{25}$$

$$\cos \beta = \sin \beta = \sqrt{1 - \cos^2 \beta} = \frac{3}{5}$$

$$\sin(\alpha - \beta) = \frac{24}{5} \cdot \left(-\frac{4}{5}\right) + \frac{7}{25} \cdot \frac{3}{5} =$$

=

2/125 96

$$p) \sin(\alpha + \beta) \quad \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin \alpha = \frac{3}{5}$$

$$\cos \beta = \frac{3}{5}$$

$$\frac{\pi}{2} < \alpha < \pi$$

$$\cos(\alpha - \beta) < 0$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\cos \beta = \sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = -\frac{4}{5} \cdot \frac{3}{5} + \frac{3}{5} \cdot \frac{4}{5} = 0$$

$$-\frac{12}{25} + \frac{12}{25} = 0$$

$$\sin \beta = \frac{4}{5} < 0$$

$$\sin \beta = \frac{4}{5} \Rightarrow \sin \beta = \frac{4}{5}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta =$$

$$= \frac{3}{5} \cdot \frac{3}{5} + \left(-\frac{4}{5}\right) \cdot \left(-\frac{4}{5}\right) = \frac{9}{25} + \frac{16}{25} = 1$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta =$$

$$= -\frac{4}{5}$$

$$\cos(\alpha + \beta) = \sqrt{1 - \sin^2(\alpha + \beta)} = 0$$

2/4/21 99

$$13x^2 - 9\sqrt{3}x + 4 = 0$$

$$\Delta = 81 \cdot 3 - 16 \cdot 13 = 243 - 208 = 35$$

$$\operatorname{tg} \ominus = -\operatorname{tg}(A+B)$$

$$\operatorname{tg}(A+B) = \frac{\operatorname{tg} A + \operatorname{tg} B}{1 - \operatorname{tg} A \operatorname{tg} B} = \frac{x_1 + x_2}{1 - x_1 x_2} =$$

$$\left\{ \begin{array}{l} x_1 + x_2 = \frac{9\sqrt{3}}{13} \\ x_1 x_2 = \frac{4}{13} \end{array} \right.$$

$$\frac{9\sqrt{3}}{13 \left(1 - \frac{4}{13}\right)} = -\sqrt{3}$$

$$\angle C = 120^\circ$$

$$\operatorname{tg} 120^\circ = \operatorname{tg}(180^\circ - 60^\circ) = -\operatorname{tg} 60^\circ = -\sqrt{3}$$

2/ упр 104.

$$9) \quad 8 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} = 8 \left(\sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right)^2 =$$

$$= 8 \cdot \left(\frac{1}{2} \sin \frac{\pi}{4} \right)^2 = 8 \cdot \frac{1}{4} \cdot \left(\frac{\sqrt{2}}{2} \right)^2 = 1.$$

$$11) \quad 2 \left(\sin^2 \frac{\pi}{8} - \cos^2 \frac{\pi}{8} \right)^2 = -2 \left(\sin^2 \frac{\pi}{8} + (-\cos^2 \frac{\pi}{8}) \right)^2 =$$

$$= 2 \left(-\cos^2 \frac{\pi}{8} \right)^2 = 2 \cdot \frac{1}{2} = 1$$

$$6) \quad 4 \sin 105^\circ \cos 105^\circ = 2 \cdot \sin 2 \cdot 105^\circ = 2 \cdot \sin 210^\circ =$$

$$= 2 \sin(180^\circ + 30^\circ) = -2 \sin(30^\circ) = -1$$

$$4) \cos^2 165^\circ - \sin^2 165^\circ = \cos 2 \cdot 165^\circ = \cos 330^\circ = \\ = \cos (360^\circ - 30^\circ) = \cos (30^\circ) = \frac{\sqrt{3}}{2}$$

2/4/1/106.

$$f) 2 \cos^2 \frac{\pi + \lambda}{4} - 2 \sin^2 \left(\frac{\pi + \lambda}{4} \right) = \cos 2\lambda + 1$$

$$\cos 2 \cdot \frac{\pi + \lambda}{4} + 1 \quad \text{---} \quad \cos 2 \cdot \frac{\pi + \lambda}{2} = 2 \cos 2 \cdot \frac{\pi + \lambda}{2}$$

$$= 2 \cos \left(\frac{\pi}{2} + \frac{\lambda}{2} \right) = -2 \sin \left(\frac{\lambda}{2} \right)$$

$$= \cos 2 \cdot \frac{\pi + \lambda}{4} + 1$$

$$7) 4 \sin \frac{a}{2} \sin \frac{\pi - \lambda}{2} \sin \frac{3\pi - 2a}{2} =$$

$$= 4 \sin \frac{a}{2} \sin \left(\frac{\pi}{2} - \frac{\lambda}{2} \right) \sin \left(\frac{3\pi}{2} - a \right) =$$

$$= 4 \sin \frac{a}{2} \cos \left(\frac{\lambda}{2} \right) (-\cos a) =$$

$$= -4 \sin \frac{a}{2} \cos \frac{\lambda}{2} \cos a = -2 \left(2 \sin \frac{a}{2} \right) \cos a =$$

$$= -2 \sin a$$

2/4/2018

$$u) \frac{\sin 2x - 2 \sin x}{\cos x - 1} = \frac{2 \sin x (\cos x - 1)}{\cos x - 1} = 2 \sin x$$

$$q) \frac{\cos 2x - \cos^2 x}{1 - \cos^2 x} = \frac{\cos^2 x - \sin^2 x - \cos^2 x}{1 - \cos^2 x \sin^2 x} = -1$$

$$t) (\cot x - \tan x) \sin 2x = \left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \right) \sin 2x =$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \cdot 2 \sin x \cos x = 2 \cos 2x$$

2/4/2018

$$u) \frac{1 + \sin 2x}{(\sin x + \cos x)^2} = \frac{1 + 2 \sin x \cos x}{1 + 2 \sin x \cos x} = 1$$

$$q) 2 \cos^2 \frac{x}{6} - 1 = \cos \frac{x}{3}$$

1/11

$$q) \frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \frac{1 - 1 - 2 \sin^2 x + 2 \cos^2 x - 1}{1 + 1 + 2 \sin^2 x + 2 \cos^2 x - 1} =$$

$$= \frac{2 \cos^2 x - 1}{3} =$$

$$q) 1 - \cos 2\alpha =$$

$$r) \frac{\cos^2 2\alpha - 4\cos^2 \alpha + 3}{\cos^2 2\alpha + 4\cos^2 \alpha - 1} =$$

$$= \frac{(2\cos^2 \alpha - 1)^2 - 4\cos^2 \alpha + 3}{(2\cos^2 \alpha - 1)^2 + 4\cos^2 \alpha - 1} =$$

$$= \frac{4\cos^4 \alpha - 4\cos^2 \alpha + 1 - 4\cos^2 \alpha + 3}{4\cos^4 \alpha - 4\cos^2 \alpha + 1 + 4\cos^2 \alpha - 1} =$$

$$= \frac{4(\cos^4 \alpha - 2\cos^2 \alpha + 1)}{4\cos^4 \alpha} = \frac{(\cos^2 \alpha - 1)^2}{\cos^2 \alpha}$$

$$\cos 2\alpha = \cos^2 \alpha - 1 + \cos^2 \alpha = 2\cos^2 \alpha - 1$$

19.02.2006p.

2/урд 6с

$$u) \sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$p) \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{ctg} \alpha$$

$$q) \cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$r) \operatorname{ctg}(\alpha - \pi) = -\operatorname{ctg}(\pi - \alpha) = \operatorname{ctg} \alpha$$

$$t) \operatorname{tg}(\alpha - \pi) = -\operatorname{tg}(\pi - \alpha) = \operatorname{tg} \alpha$$

$$y) \cos\left(\frac{3}{2}\pi - \alpha\right) = -\sin \alpha$$

$$z) \sin\left(\alpha - \frac{3}{2}\pi\right) =$$

$$= -\sin\left(\frac{3}{2}\pi - \alpha\right) = \cos \alpha$$

$$u) \cos(\pi + \alpha) = -\cos \alpha$$

$$p) \operatorname{ctg}(\pi - \alpha) = -\operatorname{ctg} \alpha$$

2/урд 66

$$\text{уш } \alpha = 210^\circ = 180^\circ + 30^\circ$$

$$\sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

$$\cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\operatorname{tg}(180^\circ + 30^\circ) = \operatorname{tg} 30^\circ = \frac{\sqrt{3}}{3}$$

$$\operatorname{ctg}(180^\circ + 30^\circ) = \operatorname{ctg} 30^\circ = \sqrt{3}$$

$$\text{А) } \alpha = \frac{5}{4}\pi = \pi + \frac{\pi}{4}$$

$$\sin\left(\pi + \frac{\pi}{4}\right) = -\sin \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\cos\left(\pi + \frac{\pi}{4}\right) = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\operatorname{tg}\left(\pi + \frac{\pi}{4}\right) = \operatorname{tg} \frac{\pi}{4} = 1$$

$$\operatorname{ctg}\left(\pi + \frac{\pi}{4}\right) = \operatorname{ctg} \frac{\pi}{4} = 1$$

$$\text{в) } \alpha = \frac{4}{3}\pi = \pi + \frac{\pi}{3}$$

$$\sin\left(\pi + \frac{\pi}{3}\right) = -\sin \frac{\pi}{3} = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\pi + \frac{\pi}{3}\right) = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$\operatorname{tg}\left(\pi + \frac{\pi}{3}\right) = \operatorname{tg} \frac{\pi}{3} = \sqrt{3}$$

$$\operatorname{ctg}\left(\pi + \frac{\pi}{3}\right) = \operatorname{ctg} \frac{\pi}{3} = \frac{\sqrt{3}}{3}$$

7) $\alpha = 300^\circ = 240^\circ + 30^\circ$

IV quadrant

$$\sin(240^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos(240^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$\operatorname{tg}(240^\circ + 30^\circ) = -\operatorname{ctg} 30^\circ = -\sqrt{3}$$

$$\operatorname{ctg}(240^\circ + 30^\circ) = -\operatorname{tg} 30^\circ = -\frac{\sqrt{3}}{3}$$

8) $\operatorname{tg}(\alpha - 2\pi)$ б) $\alpha = \frac{9}{4}\pi = 2\pi + \frac{\pi}{4}$ I quadrant

$$\sin\left(2\pi + \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\left(2\pi + \frac{\pi}{4}\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\operatorname{tg}\left(2\pi + \frac{\pi}{4}\right) = \operatorname{tg} \frac{\pi}{4} = 1$$

$$\operatorname{ctg}\left(2\pi + \frac{\pi}{4}\right) = \operatorname{ctg} \frac{\pi}{4} = 1$$

9) $\alpha = 330^\circ = 360^\circ - 30^\circ$

IV quadrant

$$\sin(360^\circ - 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

$$\cos(360^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\operatorname{tg}(360^\circ - 30^\circ) = -\operatorname{tg} 30^\circ = -\frac{\sqrt{3}}{3}$$

$$\operatorname{ctg}(360^\circ - 30^\circ) = -\operatorname{ctg} 30^\circ = -\sqrt{3}$$

з/урд 67

$$\begin{aligned} \text{м)} \operatorname{ctg}(90^\circ - \alpha) &= \operatorname{tg} \alpha \\ \text{Р)} \cos(90^\circ + \alpha) &= -\sin \alpha \\ \text{г)} \sin(270^\circ - \alpha) &= -\cos \alpha \end{aligned}$$

$$\begin{aligned} \text{н)} \sin(270^\circ + \alpha) &= -\cos \alpha \\ \text{б)} \operatorname{tg}(\alpha - 270^\circ) &= -\operatorname{ctg} \alpha \\ \text{у)} \operatorname{ctg}(\alpha - 180^\circ) &= \operatorname{ctg} \alpha \end{aligned}$$

з/урд 68

$$\begin{aligned} \text{м)} \sin \alpha &= \cos\left(\frac{\pi}{2} - \alpha\right) \\ \text{г)} \operatorname{tg} \alpha &= \operatorname{ctg}\left(\frac{\pi}{2} - \alpha\right) \end{aligned}$$

$$\begin{aligned} \text{Р)} \cos \alpha &= \sin\left(\frac{\pi}{2} - \alpha\right) \\ \text{н)} \operatorname{ctg} \alpha &= \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) \end{aligned}$$

з/урд 69.

$$\begin{aligned} \text{м)} \cos(810^\circ + \alpha) &= \cos(2 \cdot 360^\circ + 90^\circ + \alpha) = \cos(90^\circ + \alpha) = -\sin \alpha \\ \text{Р)} \sin(990^\circ - \alpha) &= \sin(2 \cdot 360^\circ + 270^\circ - \alpha) = \sin(270^\circ - \alpha) = -\cos \alpha \\ \text{г)} \operatorname{tg}(\alpha - 450^\circ) &= -\operatorname{tg}(360^\circ + 90^\circ - \alpha) = -\operatorname{tg}(90^\circ - \alpha) = \operatorname{ctg} \alpha \\ \text{н)} \operatorname{tg}(7\pi - \alpha) &= \operatorname{tg}(\pi - \alpha) = -\operatorname{tg} \alpha \end{aligned}$$

$$\begin{aligned} \text{б)} \cos\left(\alpha - \frac{13\pi}{2}\right) &= -\cos\left(\frac{\pi}{2} - \alpha\right) = -\sin \alpha \\ \text{у)} \operatorname{ctg}\left(\frac{7\pi}{2} + \alpha\right) &= \operatorname{ctg}\left(\frac{3\pi}{2} + \alpha\right) = -\operatorname{tg} \alpha \end{aligned}$$

2/урд 70

$$\text{уш } \sin^2(\pi + x) = (\sin(\pi + x))^2 = (-\sin x)^2 = \sin^2 x$$

$$F) \cos^2\left(\frac{3\pi}{2} - x\right) = \left(\cos\left(\frac{3\pi}{2} - x\right)\right)^2 = (-\sin x)^2 = \sin^2 x$$

$$G) \operatorname{tg}^2(\pi + x) = (\operatorname{tg}(\pi + x))^2 = (\operatorname{tg} x)^2 = \operatorname{tg}^2 x$$

$$H) \cos^4(\pi - x) = (\cos(\pi - x))^4 = (-\cos x)^4 = \cos^4 x$$

$$I) \sin^3\left(\frac{3\pi}{2} - x\right) = \left(\sin\left(\frac{3\pi}{2} - x\right)\right)^3 = (-\cos x)^3 = -\cos^3 x$$

$$J) \operatorname{ctg}^3\left(\frac{3\pi}{2} + x\right) = \left(\operatorname{ctg}\left(\frac{3\pi}{2} + x\right)\right)^3 = (-\operatorname{tg} x)^3 = -\operatorname{tg}^3 x$$

2/урд 71

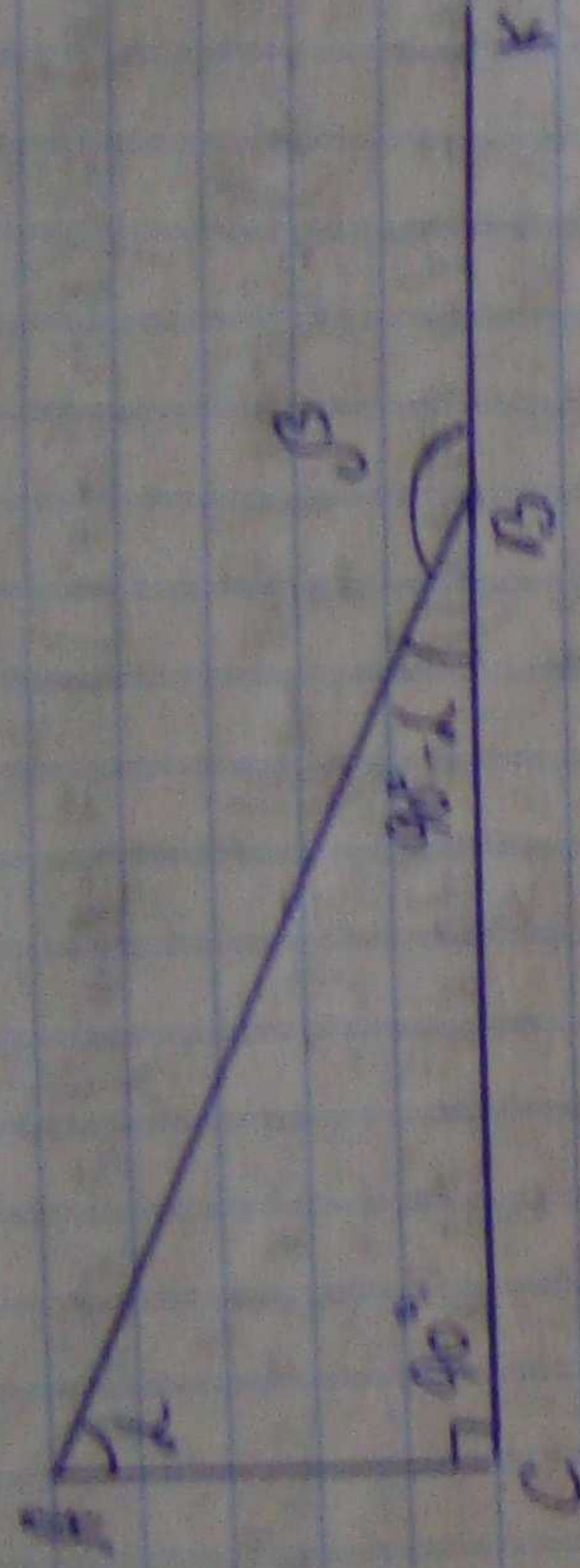
$$\text{уш } \sin^2(180^\circ - \alpha) + \sin^2(270^\circ - \alpha) = \sin^2 \alpha + \cos^2 \alpha = 1$$

$$F) \sin(90^\circ - \alpha) + \cos(180^\circ + \alpha) + \operatorname{tg}(270^\circ + \alpha) + \operatorname{ctg}(360^\circ + \alpha) = \cos \alpha - \cos \alpha - \operatorname{ctg} \alpha + \operatorname{ctg} \alpha = 0$$

$$G) \sin(\pi + \alpha) \cos\left(\frac{\pi}{2} + \alpha\right) - \cos(2\pi + \alpha) \sin\left(\frac{3\pi}{2} - \alpha\right) = -\sin \alpha \cdot (-\sin \alpha) - \cos \alpha (-\cos \alpha) = \sin^2 \alpha + \cos^2 \alpha = 1$$

$$H) \cos(\pi + \alpha) \sin\left(\frac{\pi}{2} + \alpha\right) + \sin(2\pi + \alpha) \cos\left(\frac{3\pi}{2} + \alpha\right) = -\cos \alpha \cdot \cos \alpha + \sin \alpha \cdot \sin \alpha = \sin^2 \alpha - \cos^2 \alpha$$

Задание 72



$$\operatorname{tg}(\angle ABC) = -\sqrt{5}$$

$$\operatorname{tg}(\angle ABC) = \operatorname{tg}(\angle CAB) \text{ or } \operatorname{tg}(\angle ABC) = 2$$

$$(\operatorname{tg}(\angle ABC) = \operatorname{tg}(90^\circ + \angle CAB))$$

$$\operatorname{tg}(\beta) = -\sqrt{5}$$

$$\operatorname{tg}(\alpha) = \operatorname{tg}(90^\circ - \beta) = 2$$

$$\operatorname{tg} \beta = \operatorname{tg}(90^\circ + \alpha) = -\operatorname{ctg} \alpha = -\sqrt{5}$$

$$\operatorname{ctg} \alpha = \sqrt{5} \Rightarrow \operatorname{tg} \alpha = \frac{\sqrt{5}}{5}$$

$$\operatorname{tg}(90^\circ - \alpha) = \operatorname{ctg} \alpha = \sqrt{5}$$

$$\text{Итого: } \operatorname{tg} \alpha = \frac{\sqrt{5}}{5} \text{ or } \operatorname{tg}(90^\circ - \alpha) = \sqrt{5}$$

Задание 73

$$\text{или } \operatorname{tg} \alpha = 2 \text{ or } \sin(2\alpha - \alpha) = 2, \operatorname{tg} \alpha = \frac{\sqrt{5}}{5} \text{ or } \frac{\sqrt{5}}{5}$$

$$\pi - \alpha < \frac{3}{2}\pi, \operatorname{tg} \alpha = \frac{\sqrt{5}}{5} \in (-\infty, \infty) \text{ or } \operatorname{tg} \alpha = 2$$

$$27 \sin(2\pi - \alpha) = 27 \sin \alpha$$

$$\cos \alpha = -\frac{\sqrt{65}}{9} \Rightarrow \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{65}{81}}$$

$$= \sqrt{\frac{16}{81}} = \frac{4}{9}$$

$$\begin{cases} \sin \alpha = \pm \frac{4}{9} \\ \alpha \in [0, \pi] \end{cases} \Rightarrow \sin \alpha = -\frac{4}{9}$$

$$-27 \sin \alpha = 27 \cdot \frac{4}{9} = 12$$

$$\text{Answer: } 27 \sin(2\pi - \alpha) = 12$$

$$P) 27 \cos\left(\frac{3\pi}{2} - \alpha\right) = 27 \cos \alpha = -\frac{\sqrt{32}}{9} \quad \text{and } \frac{\pi}{2} < \alpha < \pi$$

$$\cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin \alpha$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{32}{81}} = \sqrt{\frac{49}{81}} = \frac{7}{9}$$

$$\sin \alpha = \frac{7}{9}$$

$$27 \cos\left(\frac{3\pi}{2} - \alpha\right) = -27 \sin \alpha = -27 \cdot \frac{7}{9} = -21$$

Answer: -21

$$7) \cos \alpha = -\frac{\sqrt{21}}{5} \quad \text{and } \alpha < \frac{3}{2}\pi$$

$$\text{Amplitude } 15 \cos\left(\frac{3\pi}{2} + \alpha\right) = 2$$

$$\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin \alpha$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{21}{25}} = \sqrt{\frac{4}{25}} = \pm \frac{2}{5}$$

$$\sin \alpha = -\frac{2}{5}$$

$$15 \cos\left(\frac{3\pi}{2} + \alpha\right) = 15 \sin \alpha = 15 \cdot \frac{2}{5} = 6$$

$$\eta_{\text{exp}} = -6$$

$$\eta) 3,45 \sin\left(\frac{3\pi}{2} + \alpha\right), \text{ t.p.k. } \sin \alpha = \frac{3}{5} \quad \text{to } 0 < \alpha < \frac{\pi}{2}$$

$$\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos \alpha$$

$$\sin \cos \alpha = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \pm \frac{4}{5}$$

$$\cos \alpha = \frac{4}{5}$$

$$3,45 \sin\left(\frac{3\pi}{2} + \alpha\right) = -3,45 \cos \alpha = -3,45 \cdot \frac{4}{5} = -2,76$$

$$\eta_{\text{exp}} = -3$$

2. упр. 74

$$\text{у)} \frac{2 \sin \alpha - \cos \alpha}{\sin \alpha - 2 \cos \alpha} = \frac{2 \sqrt{1 - \cos^2 \alpha} - \cos \alpha}{\sqrt{1 - \cos^2 \alpha} - 2 \cos \alpha}$$

$$= \frac{2 \cdot \operatorname{tg} \alpha \cdot \cos \alpha - \cos \alpha}{\operatorname{tg} \alpha \cos \alpha - 2 \cos \alpha} = \frac{\cos \alpha (2 \operatorname{tg} \alpha - 1)}{\cos \alpha (\operatorname{tg} \alpha - 2)}$$

$$\frac{2 \operatorname{tg} x - 1}{\operatorname{tg} x - 2} = 3$$

$$2 \operatorname{tg} x - 1 = 3 \operatorname{tg} x - 6$$

$$\operatorname{tg} x = 5$$

Проверка: 5

$$A) \frac{5 \sin x - 2 \cos(\pi + x)}{\sin(\pi - x) + 3 \cos x} = 3$$

$$\frac{5 \sin x + 2 \cos x}{\sin x + 3 \cos x} = \frac{5 \operatorname{tg} x \cos x + 2 \cos x}{\operatorname{tg} x \cos x + 3 \cos x} = \frac{\cos x (5 \operatorname{tg} x + 2)}{\cos x (\operatorname{tg} x + 3)}$$

$$\frac{5 \operatorname{tg} x + 2}{\operatorname{tg} x + 3} = 3$$

$$5 \operatorname{tg} x + 2 = 3 \operatorname{tg} x + 9$$

$$2 \operatorname{tg} x = 7$$

$$\operatorname{tg} x = 3,5$$

$$\text{Проверка: } \operatorname{tg} x = 3,5$$

$$\text{Упрощение: } \operatorname{ctg} x = 2$$

Проверка: 2

$$B) \frac{2 \sin x + \cos x}{\sin x + 2 \cos x} = 4$$

$$\frac{2 \sin x + \cos x}{\sin x + 2 \cos x} = \frac{2 \sin x + \operatorname{ctg} x \sin x}{\sin x + 2 \operatorname{ctg} x \sin x} = \frac{2 + \operatorname{ctg} x}{1 + 2 \operatorname{ctg} x}$$

$$\frac{2 + \operatorname{ctg} x}{1 + 2 \operatorname{ctg} x} = 4$$

$$2 + \operatorname{ctg} x = 4 + 8 \operatorname{ctg} x$$

$$7 \operatorname{ctg} x = -2 \quad \operatorname{ctg} x = -\frac{2}{7}$$

$$P) \frac{6 \cos\left(\frac{3\pi}{2} + x\right) - \cos x}{\sin x - 3 \cos(\pi - x)} = 5$$

$$\frac{6 \sin x - \cos x}{\sin x + 3 \cos x} = \frac{6 \sin x - \cos x \sin x}{\sin x + 3 \cos x \sin x} = \frac{6 - \cos x}{1 + 3 \cos x} = 5$$

$$6 - \cos x = 5 + 15 \cos x$$

$$15 \cos x = 1$$

$$\cos x = \frac{1}{15}$$

$$\arccos \frac{1}{15}$$

2/2/2017

Quadratic

$$\sin x = 2 \sin^2 x + 5 \sin x + 2 = 0$$

$$2 \sin^2 x + 5 \sin x + 2 = 0$$

$$\sin x_{1,2} = \frac{-5 \pm 3}{4} = -\frac{1}{2}$$

$$\sin x = -\frac{1}{2} \quad \arcsin \left(-\frac{1}{2}\right)$$

$$P) 2 \sin^2 x - 5 \sin x + 2 = 0$$

$$2 \sin^2 x - 5 \sin x + 2 = 0$$

$$\sin x_{1,2} = \frac{5 \pm 3}{4} = \frac{1}{2} \quad \arcsin \frac{1}{2}$$

$$9) \cos^2 \lambda + 1 = 2 \cos \left(\frac{\pi}{2} + \lambda \right)$$

$$\cos^2 \lambda + 1 = -2 \sin \lambda$$

$$-\cos^2 \lambda + 1 = -2$$

$$1 - \sin^2 \lambda + 1 = -2 \sin \lambda$$

$$\sin^2 \lambda - 2 \sin \lambda - 2 = 0$$

$$\Delta = 4 + 2 = 3$$

$$\sin \lambda_{1,2} = \frac{1 \pm \sqrt{3}}{2} \quad \begin{matrix} 1 + \sqrt{3} > 2 \\ 1 - \sqrt{3} < 0 \end{matrix} \quad \text{chpuy.}$$

$$\sin \lambda = 1 - \sqrt{3}$$

$$\text{Try: } 1 - \sqrt{3}:$$

$$1) \cos^2 \left(\frac{\pi}{2} + \lambda \right) - 2 \sin \lambda + 1 = 0$$

$$\sin^2 \lambda - 2 \sin \lambda + 1 = 0$$

$$(\sin \lambda - 1)^2 = \sin^2 \lambda - 2 \sin \lambda + 1 = 0$$

$$\sin \lambda = 1.$$

2/урд 77.

Решать $\cos \lambda = 2$, брб

$$\text{мы } 3 \cos^2 \lambda - 10 \cos \lambda + 3 = 0$$

$$\Delta = 25 - 9 = 16$$

$$\cos \lambda_{1,2} = \frac{5 \pm 4}{3} \quad \begin{matrix} 1 \\ 3 \end{matrix} \quad \text{чрр.}$$

$$\cos \lambda = \frac{1}{3}$$

$$A) 3 \cosh^2 \lambda + 10 \cosh \lambda + 3 = 0$$

$$b = 25 - 9 = 16$$

$$\cosh_{1,2} z = \frac{-5 \pm 4}{3} < -\frac{1}{3} \quad -3 \text{ th real}$$

$$\cosh z = -\frac{1}{3}$$

$$g) 2 \cosh(3\pi + \lambda) + \sinh^2 \lambda = 0$$

$$-2 \cosh \lambda + \sinh^2 \lambda = 0$$

$$1 - \cosh^2 \lambda - 2 \cosh \lambda = 0$$

$$\cosh^2 \lambda + 2 \cosh \lambda - 1 = 0$$

$$b = 4 - 4 = 0$$

$$\cosh z = \frac{-2}{2} z = -1$$

$$n) 2 \cosh \lambda + \sinh^2(5\pi + \lambda) = 0$$

$$2 \cosh \lambda + \sinh^2(\pi + \lambda) = 0$$

$$2 \cosh \lambda + 1 - \cosh^2 \lambda = 0$$

$$\cosh^2 \lambda - 2 \cosh \lambda - 1 = 0$$

$$b = 4 + 1 = 2$$

$$\cosh_{1,2} z = 1 \pm \sqrt{2} \quad \sqrt{1-1} = 0$$

$$\cosh z = 1 - \sqrt{2}$$

2/урок 78

$$\text{Гиперб. } \sqrt{\tanh \lambda} = -6, \text{ брб}$$

$$u) 3 \tanh^2 \lambda - 11 \tanh \lambda - 4 = 0$$

$$b = 121 + 48 = 169$$

$$\tanh_{1,2} z = \frac{11 \pm 13}{3} < -\frac{2}{9} \text{ th real}$$

$$\tanh \sqrt{2}, \sqrt{\tanh \lambda} = \sqrt{2,6} \approx 1,6, z = \sqrt{4} = 2$$

$$p) \operatorname{tg}^2 x + 2 \operatorname{tg} x - 3 = 0.$$

$$b = 1 + 3 = 4$$

$$\operatorname{tg} x_{1,2} = -1 \pm 2 = \begin{matrix} -3 \\ 1 \end{matrix}$$

$$\sqrt{\operatorname{tg} x} = 1$$

$$\sqrt{\operatorname{tg} x} = 1.$$

*)

2/урок 80

Перебери $\operatorname{ctg} x = 0$, $\operatorname{tg} x = 0$

$$u) \operatorname{ctg}^2 x - 6 \operatorname{ctg} x - 7 = 0 \quad b \quad |\operatorname{ctg} x + 1| < 2$$

$$\frac{b}{4} = 9 + 7 = 16$$

$$\operatorname{ctg} x_{1,2} = 3 \pm 4 = \begin{matrix} 7 \\ -1 \end{matrix}$$

$$|\operatorname{ctg} x + 1| < 2$$

$$\operatorname{ctg} x + 1 = 0$$

$$\operatorname{ctg} x = -1$$

$$\operatorname{ctg} x \leq -1$$

$$1 - \operatorname{ctg} x < 2$$

$$\Rightarrow \left\{ \operatorname{ctg} x \in (-\infty; -1] \right.$$

$$\left. \operatorname{ctg} x \in (3; +\infty) \right\}$$

$$\Rightarrow \operatorname{ctg} x \in \emptyset$$

$$\left\{ \operatorname{ctg} x \geq -1 \right.$$

$$\left. \operatorname{ctg} x + 1 < 2 \right\}$$

$$\Rightarrow \left\{ \operatorname{ctg} x \in [-1; +\infty) \right.$$

$$\left. \operatorname{ctg} x \in (-1 \right.$$

$$(\operatorname{ctg} x + 1)^2 < 4$$

$$\operatorname{ctg}^2 x + 2 \operatorname{ctg} x - 3 = 0$$

$$b = 1 + 3 = 4$$

$$\operatorname{ctg} x_{1,2} = -1 \pm 2 = \begin{matrix} -3 \\ 1 \end{matrix}$$

$$ctg \lambda \in (-3; 1)$$

$$ctg \lambda = -1.$$

$$P) ctg^2 \lambda - 3ctg \lambda - 10 = 0 \quad k \quad |ctg \lambda - 3| < 3$$

$$b = 9 + 40 = 49$$

$$ctg \lambda_{1,2} = \frac{3 \pm \sqrt{49}}{2} = \frac{-2}{5}$$

$$|ctg \lambda - 3| < 3$$

$$(ctg \lambda - 3)^2 < 9$$

$$ctg^2 \lambda - 6ctg \lambda = 0$$

$$ctg \lambda (ctg \lambda - 6) = 0$$

$$ctg \lambda = 0; 6$$

$$ctg \lambda \in (0; 6)$$

$$ctg \lambda$$

$$\begin{cases} ctg \lambda = -2; 5 \\ ctg \lambda \in (0; 6) \end{cases} \Rightarrow ctg \lambda = 5$$

$$\sin \lambda ctg \lambda = 5$$

Результат 82

$$s = 2845$$

$$t = 171$$

$$V_q = 34571$$

$$V_u = ?$$

Temp to approx - 0

96266 *Hamamelis virginica*

ms - S-575-pr

$$\left(\frac{28}{x+3} \right)^d$$

$$\begin{array}{r} 28 \\ x+3 \end{array} + \begin{array}{r} 28 \\ x-3 \end{array} = 7.$$

$$\frac{28(x-3+x+3)}{(x^2-3)^2 \cdot x^2-9} \cdot x$$

$$2x \cdot 28 = 4x^2 - 42x + 63$$

$$4x^2 - 98x + 63 = 0$$

$$x^2 - 14x + 9 = 0$$

$$\frac{5}{2} \times 49 - 9 = 40$$

$$x_1, x_2 = y \pm \sqrt{y^2 - 40} = 7 \pm \sqrt{49 - 40} = 7 \pm \sqrt{9} = 7 \pm 3 = 4, 10$$

$$\frac{28 - 2x^2}{x^2}$$

$x^2 - 9$

$$B_2 X_2 X_8$$

$$x^2 - 8x - 9 = 0$$

$$\frac{b}{4} \quad 2 \quad 16 + 9 = 25$$

$$x_{12} z + y \pm s -$$

9/5/76: well

$$\left\{ \begin{array}{l} \text{Zeilvertauschung} \\ \text{neutrale Spalte} \\ \left(\frac{28}{x-3} \right) d - 28 \end{array} \right. \quad \left\{ \begin{array}{l} \text{28 Spalten} \\ 7 d - 28 \end{array} \right.$$

$$\left(\frac{28}{x-3}\right)^{1-225}$$

Julius 83.

Bewertung nach 64-2 = 8045.

Bewertung 4-42 = 96.

Bewertung nach 4-42 = 18457d.

Wertung nach 4-42 = X

Wertung nach 4-42 = $\left(\frac{90}{x+18}\right)$ fms

Wertung nach 4-42 = $\left(\frac{90}{18-x}\right)$ fms

$$\frac{90}{x+18} + \frac{90}{x-18} = 9$$

$$90 \left(\frac{1}{x+18} + \frac{1}{18-x} \right) = 9$$

$$90 \cdot 2x = 9(x^2 - 18^2)$$

$$10 \left(\frac{36}{18^2 - x^2} \right) = 9$$

$$10 \cdot 2x = x^2 - 18^2$$

$$10 \cdot 36 = 18^2 - x^2$$

$$x^2 - 20x - 324 = 0$$

$$x^2 - 324 + 360 = 0$$

$$\frac{x}{4} = 100 + 324 = 424$$

$$x^2 + 36 = 0$$

$$x^2 = 36$$

$$x_1, 2 = 10 \pm$$

24 fms

Freitag 84

9.5.18. 15.45. 15.45. 15.45. 15.45.

15.45.

15.45. 15.45. 15.45. 15.45. 15.45.

15.45. 15.45. 15.45. 15.45. 15.45.

15.45. 15.45. 15.45. 15.45. 15.45.

15.45. 15.45. 15.45. 15.45. 15.45.

$$\frac{15}{x} - \frac{6}{x-2} = 1$$

$$\frac{15(x-2) - 6x}{x(x-2)} = 1$$

$$15x - 30 - 6x = x^2 - 2x$$

$$x^2 - 11x + 30 = 0$$

$$\Delta = 121 - 120 = 1$$

$$x_{1,2} = \frac{11 \pm 1}{2} = \frac{12}{2} = 6 \quad \frac{10}{2} = 5$$

Freitag 85

$$\cos\left(\frac{\pi}{4} - \alpha\right) = \left(\cos\alpha\cos\frac{\pi}{4} + \sin\alpha\sin\frac{\pi}{4}\right) \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\sin\alpha =$$

$$= \frac{\sqrt{2}}{2} (\cos\alpha + \sin\alpha)$$

$$f) \cos\left(\frac{\pi}{4} + \alpha\right) = \cos\frac{\pi}{4}\cos\alpha - \sin\frac{\pi}{4}\sin\alpha = \frac{\sqrt{2}}{2} (\cos\alpha - \sin\alpha)$$

$$q) \sin\left(\frac{\pi}{4} - \alpha\right) = \sin \frac{\pi}{4} \cos \alpha - \cos \frac{\pi}{4} \sin \alpha = \frac{\sqrt{2}}{2} (\cos \alpha - \sin \alpha)$$

$$r) \sin\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \sin \alpha = \frac{\sqrt{2}}{2} (\cos \alpha + \sin \alpha)$$

$$h) \operatorname{tg}\left(\frac{\pi}{4} + \alpha\right) = \frac{\operatorname{tg} \frac{\pi}{4} + \operatorname{tg} \alpha}{1 - \operatorname{tg} \frac{\pi}{4} \operatorname{tg} \alpha} =$$

$$= \frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha}$$

$$y) \operatorname{tg}\left(\frac{\pi}{4} - \alpha\right) = \frac{\operatorname{tg} \frac{\pi}{4} - \operatorname{tg} \alpha}{1 + \operatorname{tg} \frac{\pi}{4} \operatorname{tg} \alpha} = \frac{1 - \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha}$$

Jumped 86

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \sin \frac{\pi}{2} \cos \alpha + \cos \frac{\pi}{2} \sin \alpha = \cos \alpha \neq 0 \neq \cos \alpha$$

$$\cos\left(\frac{\pi}{2} + \alpha\right) = \cos \frac{\pi}{2} \cos \alpha - \sin \frac{\pi}{2} \sin \alpha = -\sin \alpha$$

$$\sin(\pi + \alpha) = \sin \pi \cos \alpha + \cos \pi \sin \alpha = 0 - \sin \alpha = -\sin \alpha$$

$$\cos(\pi + \alpha) = \cos \pi \cos \alpha - \sin \pi \sin \alpha = -\cos \alpha$$

$$\sin\left(\frac{3\pi}{2} + \alpha\right) = \sin \frac{3\pi}{2} \cos \alpha + \cos \frac{3\pi}{2} \sin \alpha =$$

$$= -\cos \alpha$$

$$\cos\left(\frac{3\pi}{2} + \alpha\right) = \cos \frac{3\pi}{2} \cos \alpha - \sin \frac{3\pi}{2} \sin \alpha =$$

$$= \sin \alpha$$

2/уезд 87.

$$\text{у)} \quad \sqrt{2} \sin\left(\frac{\pi}{4} + \alpha\right) - \sin \alpha = \sqrt{2} \cdot \frac{\sqrt{2}}{2} \cdot (\sin \alpha + \cos \alpha) - \sin \alpha =$$

$$= \cos \alpha$$

$$\text{р)} \quad \sqrt{2} \cos\left(\frac{\pi}{4} - \alpha\right) = \sqrt{2} \cdot \frac{\sqrt{2}}{2} (\sin \alpha + \cos \alpha) - \cos \alpha = \sin \alpha$$

$$\text{г)} \quad 2 \sin\left(\frac{\pi}{6} + \alpha\right) - \cos \alpha = 2 \left(\sin \frac{\pi}{6} \cos \alpha + \cos \frac{\pi}{6} \sin \alpha \right) - \cos \alpha =$$

$$= 2 \left(\frac{1}{2} \cdot \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right) - \cos \alpha = \cos \alpha + \sqrt{3} \sin \alpha - \cos \alpha =$$

$$= \sqrt{3} \sin \alpha$$

$$\text{н)} \quad \sqrt{2} \cos \alpha - 2 \cos \alpha \left(\frac{\pi}{4} + \alpha \right) = \sqrt{2} \cos \alpha - (\cos \alpha + \sin \alpha) = \sqrt{2} \sin \alpha$$

2/уезд 88

$$\text{у)} \quad \frac{\sqrt{2} \cos\left(\frac{3\pi}{4} + \alpha\right) + \cos \alpha}{\sqrt{2} \cos\left(\frac{5\pi}{4} - \alpha\right) + \sin \alpha} = \frac{\sqrt{2} \left(\cos \frac{3\pi}{4} \cos \alpha - \sin \frac{3\pi}{4} \sin \alpha \right) + \cos \alpha}{\sqrt{2} \left(\cos \frac{5\pi}{4} \cos \alpha + \sin \frac{5\pi}{4} \sin \alpha \right) + \sin \alpha} =$$

$$= \frac{\sqrt{2} \left(\frac{\sqrt{2}}{2} (\cos \alpha - \sin \alpha) \right) + \cos \alpha}{\sqrt{2} \left(-\frac{\sqrt{2}}{2} (\cos \alpha + \sin \alpha) \right) + \sin \alpha} = \frac{2 \cos \alpha - \sin \alpha}{\cos \alpha + 2 \sin \alpha}$$

$$= \frac{\operatorname{tg} \alpha \cdot \cos \alpha - 2 \cos \alpha}{2 \operatorname{tg} \alpha \cos \alpha + \cos \alpha} = \frac{\cos \alpha (\operatorname{tg} \alpha - 2)}{\cos \alpha (2 \operatorname{tg} \alpha + 1)} = \frac{\operatorname{tg} \alpha - 2}{2 \operatorname{tg} \alpha + 1}$$

$$\begin{aligned}
 p) \frac{\sin(\frac{2\pi}{3} + \lambda) + \frac{1}{2} \sin \lambda}{\sin(\frac{7\pi}{6} - \lambda) + \frac{1}{2} \cos \lambda} &= \frac{\frac{\sqrt{3}}{2} \cos \lambda + \frac{1}{2} \sin \lambda + \frac{1}{2} \sin \lambda}{-\frac{1}{2} \cos \lambda + \frac{\sqrt{3}}{2} \sin \lambda + \frac{1}{2} \cos \lambda} = \\
 &= \frac{\frac{\sqrt{3}}{2} \cos \lambda + \sin \lambda}{-\frac{1}{2} \cos \lambda + \frac{\sqrt{3}}{2} \sin \lambda} = \frac{\sin \lambda \operatorname{ctg} \lambda \cdot \frac{\sqrt{3}}{2} + \sin \lambda}{\frac{\sqrt{3}}{2} \sin \lambda} =
 \end{aligned}$$

$$= \left(\frac{\sqrt{3}}{2} \operatorname{ctg} \lambda + 1 \right) \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{3} \operatorname{ctg} \lambda + 2}{\sqrt{3}} = \operatorname{ctg} \lambda + \frac{2\sqrt{3}}{3}$$

2/урд 89

$$\text{мы } \sin 27^\circ \cos 3^\circ + \cos 27^\circ \sin 3^\circ = \sin(27^\circ + 3^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$p) \cos 87^\circ \cos 24^\circ + \sin 87^\circ \sin 24^\circ = \cos(87^\circ - 24^\circ) = \cos 60^\circ = \frac{1}{2}$$

$$q) \cos 92^\circ \cos 28^\circ - \sin 92^\circ \sin 28^\circ = \cos(92^\circ + 28^\circ) = \cos 120^\circ = -\frac{1}{2}$$

$$r) \sin 105^\circ \cos 45^\circ - \cos 105^\circ \sin 45^\circ = \sin(105^\circ - 45^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

2/урд 90

$$\text{мы } \left(\sin \frac{\pi}{15} + \cos \frac{\pi}{10} \right)^2 + \left(\cos \frac{\pi}{15} + \sin \frac{\pi}{10} \right)^2 = 3$$

$$\sin^2 \frac{\pi}{15} + 2 \sin \frac{\pi}{15} \cos \frac{\pi}{10} + \cos^2 \frac{\pi}{10} + \cos^2 \frac{\pi}{15} + 2 \cos \frac{\pi}{15} \sin \frac{\pi}{10} + \sin^2 \frac{\pi}{10} =$$

$$= 2 \left(\sin^2 \frac{\pi}{15} + \cos^2 \frac{\pi}{15} \right) + \left(\cos^2 \frac{\pi}{10} + \sin^2 \frac{\pi}{10} \right) + 2 \left(\sin \frac{\pi}{15} \cos \frac{\pi}{10} + \cos \frac{\pi}{15} \sin \frac{\pi}{10} \right) =$$

$$= 2 + 2 \left(\sin \left(\frac{\pi}{15} + \frac{\pi}{10} \right) \right) = 2 \left(1 + \sin \left(\frac{5\pi}{30} \right) \right) = 2 \left(1 + \sin \frac{\pi}{6} \right) =$$

$$= 2 \left(1 + \frac{1}{2} \right) = 3$$

F) Чыу. Чыу чыу чыу чыу

$$\left(\cos \frac{\pi}{g} - \cos \frac{2\pi}{g}\right)^2 + \left(\sin \frac{\pi}{g} + \sin \frac{2\pi}{g}\right)^2 = 1$$

$$\cos^2 \frac{\pi}{g} - 2 \cos \frac{\pi}{g} \cos \frac{2\pi}{g} + \cos^2 \frac{2\pi}{g} + \sin^2 \frac{\pi}{g} + 2 \sin \frac{\pi}{g} \sin \frac{2\pi}{g} + \sin^2 \frac{2\pi}{g} =$$

$$= 2 + \left(\cos^2 \frac{\pi}{g} + \sin^2 \frac{\pi}{g}\right) + \left(\cos^2 \frac{2\pi}{g} + \sin^2 \frac{2\pi}{g}\right) +$$

$$+ 2 \left(\sin \frac{\pi}{g} \sin \frac{2\pi}{g} - \cos \frac{\pi}{g} \cos \frac{2\pi}{g} \right) = 2 + 2 \left(\cos \left(\frac{\pi}{g} + \frac{2\pi}{g} \right) \right) =$$

$$= 2 - 2 \cos \frac{\pi}{3} = 2 - 2 \cdot \frac{1}{2} = 2 - 1 = 1$$

чыу чыу чыу

$$\text{чыу} \frac{\cos 23^\circ - \lg 22^\circ \sin 23^\circ}{\sin 8^\circ + \lg 22^\circ \cos 8^\circ} = \sqrt{2}$$

$$\frac{\cos 23^\circ - \lg 22^\circ \cdot \lg 23^\circ \cdot \cos 23^\circ}{\sin 8^\circ + \lg 22^\circ \cdot \lg 8^\circ \cdot \sin 8^\circ} = \frac{\cos 23^\circ (1 - \lg 22^\circ \lg 23^\circ)}{\sin 8^\circ (1 + \lg 22^\circ \lg 8^\circ)}$$

?

2/4/2021

$$u) \sin(\alpha + \beta) \sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$$

$$\sin(\alpha + \beta) \sin(\alpha - \beta) = (\sin \alpha \cos \beta + \cos \alpha \sin \beta)(\sin \alpha \cos \beta - \cos \alpha \sin \beta)$$

$$= \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta =$$

$$= \sin^2 \alpha (1 - \sin^2 \beta) - (1 - \sin^2 \alpha) \sin^2 \beta =$$

$$= \sin^2 \alpha - \sin^2 \alpha \sin^2 \beta - \sin^2 \beta + \sin^2 \alpha \sin^2 \beta =$$

$$= \sin^2 \alpha - \sin^2 \beta$$

$$p) \sin(\alpha + \beta) \sin(\alpha - \beta) = \cos^2 \beta - \cos^2 \alpha$$

$$\sin(\alpha + \beta) \sin(\alpha - \beta) = (\sin \alpha \cos \beta + \cos \alpha \sin \beta)$$

$$(\sin \alpha \cos \beta - \cos \alpha \sin \beta) = \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta =$$

$$= (1 - \cos^2 \alpha) \cos^2 \beta - \cos^2 \alpha (1 - \cos^2 \beta) =$$

$$= \cos^2 \beta - \cos^2 \alpha \cos^2 \beta - \cos^2 \alpha + \cos^2 \alpha \cos^2 \beta =$$

$$= \cos^2 \beta - \cos^2 \alpha$$

$$q) \cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$$

$$(\cos \alpha \cos \beta - \sin \alpha \sin \beta)(\cos \alpha \cos \beta + \sin \alpha \sin \beta) =$$

$$= \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta = \cos^2 \alpha - \cos^2 \alpha \sin^2 \beta - \sin^2 \alpha \sin^2 \beta =$$

$$= \cos^2 \alpha - \sin^2 \beta$$

$$r) \cos(\alpha + \beta) \cos(\alpha - \beta) = (\cos \alpha \cos \beta - \sin \alpha \sin \beta)(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta = \cos^2 \beta - \sin^2 \alpha \cos^2 \beta$$

$$= -(\sin^2 \alpha \sin^2 \beta - \cos^2 \alpha \cos^2 \beta) = ?$$

2/урд 92

$$u) \sin(\alpha + \beta) \sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$$

$$\sin(\alpha + \beta) \sin(\alpha - \beta) = (\sin \alpha \cos \beta + \cos \alpha \sin \beta)(\sin \alpha \cos \beta - \cos \alpha \sin \beta)$$

$$= \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta =$$

$$= \sin^2 \alpha (1 - \sin^2 \beta) - (1 - \sin^2 \alpha) \sin^2 \beta =$$

$$= \sin^2 \alpha - \sin^2 \alpha \sin^2 \beta - \sin^2 \beta + \sin^2 \alpha \sin^2 \beta =$$

$$= \sin^2 \alpha - \sin^2 \beta$$

$$p) \sin(\alpha + \beta) \sin(\alpha - \beta) = \cos^2 \beta - \cos^2 \alpha$$

$$\sin(\alpha + \beta) \sin(\alpha - \beta) = (\sin \alpha \cos \beta + \cos \alpha \sin \beta)$$

$$\cdot (\sin \alpha \cos \beta - \cos \alpha \sin \beta) = \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta =$$

$$= (1 - \cos^2 \alpha) \cos^2 \beta - \cos^2 \alpha (1 - \cos^2 \beta) =$$

$$= \cos^2 \beta - \cos^2 \alpha \cos^2 \beta - \cos^2 \alpha + \cos^2 \alpha \cos^2 \beta =$$

$$= \cos^2 \beta - \cos^2 \alpha$$

$$q) \cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$$

$$(\cos \alpha \cos \beta - \sin \alpha \sin \beta)(\cos \alpha \cos \beta + \sin \alpha \sin \beta) =$$

$$= \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta = \cos^2 \alpha - \cos^2 \alpha \sin^2 \beta - \sin^2 \alpha \sin^2 \beta =$$

$$= \cos^2 \alpha - \sin^2 \beta$$

$$r) \cos(\alpha + \beta) \cos(\alpha - \beta) = (\cos \alpha \cos \beta - \sin \alpha \sin \beta)(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta = \cos^2 \beta - \sin^2 \alpha \cos^2 \beta$$

$$= -(\sin^2 \alpha \sin^2 \beta - \cos^2 \alpha \cos^2 \beta) = ?$$

$$u) \sin 75^\circ =$$

$$= \frac{\sqrt{2}}{2} \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right)$$

$$p) \cos 75^\circ =$$

$$= \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right)$$

$$q) \tan 15^\circ =$$

$$= \frac{\frac{\sqrt{2}}{2} - \frac{1}{2}}{1 + \frac{\sqrt{2}}{2}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} =$$

$$r) \cotg 165^\circ =$$

$$= \frac{-\frac{\sqrt{3}}{3} - 1}{1 + \frac{\sqrt{3}}{3}}$$

$$b) \sin 165^\circ =$$

$$= \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right)$$

2/урд 93

$$\text{мы } \sin 75^\circ = \sin (30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ = \frac{\sqrt{2}}{2} \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right) = \frac{\sqrt{2}(1+\sqrt{3})}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$p) \cos 75^\circ = \cos (45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) = \frac{\sqrt{2}(\sqrt{3}-1)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$q) \operatorname{tg} 15^\circ = \operatorname{tg} (45^\circ - 30^\circ) = \frac{\operatorname{tg} 45^\circ - \operatorname{tg} 30^\circ}{1 + \operatorname{tg} 45^\circ \operatorname{tg} 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

$$r) \operatorname{ctg} 165^\circ = \operatorname{ctg} (120^\circ + 45^\circ) = \frac{\operatorname{ctg} 120^\circ \operatorname{ctg} 45^\circ - 1}{\operatorname{ctg} 45^\circ + \operatorname{ctg} 120^\circ} = \frac{-\frac{1}{\sqrt{3}} - 1}{1 - \frac{1}{\sqrt{3}}} = \frac{-\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{-(1+\sqrt{3})}{\sqrt{3}-1} = \frac{(1+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{1+\sqrt{3}+\sqrt{3}+3}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

$$s) \sin 165^\circ = \sin (120^\circ + 45^\circ) = \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$t) \cos 165^\circ = \cos (120^\circ + 45^\circ) = \cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ = \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}+\sqrt{6}}{4}$$

$$u) \operatorname{tg} 165^\circ = \operatorname{tg} (120^\circ + 45^\circ) = \frac{\operatorname{tg} 120^\circ \operatorname{tg} 45^\circ + 1}{\operatorname{tg} 45^\circ - \operatorname{tg} 120^\circ} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(1-\sqrt{3})(\sqrt{3}+1)} = \frac{3+2\sqrt{3}+1}{1-3} = \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$$

2/урд 93

$$v) \sin 165^\circ = \sin (120^\circ + 45^\circ) = \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

2/урд 93

2/10/2024

auf 4. Aufgabe $\arg\left(\frac{\pi}{4} + i\right) = \frac{\pi}{4}$, $\arg\left(\frac{\pi}{4} - i\right) = -\frac{\pi}{4}$

4 $\pi < \alpha < \frac{3}{2}\pi$

$$\arg\left(\frac{\pi}{4} + i\right) = \frac{\sin \frac{\pi}{4} \cos \alpha + \cos \frac{\pi}{4} \sin \alpha}{\cos \frac{\pi}{4} \cos \alpha - \sin \frac{\pi}{4} \sin \alpha}$$

$\sin \alpha = \frac{3}{10}$ $\alpha \in \text{III}$ $\cos \alpha = -\frac{4}{10}$

$$\arg\left(\frac{\pi}{4} + i\right) = \sqrt{1 + \frac{9}{100}} = \sqrt{\frac{109}{100}} = \frac{\sqrt{109}}{10}$$

$$\arg\left(\frac{\pi}{4} + i\right) = \frac{\sqrt{2} \cdot \left(-\frac{9}{10}\right) + \frac{\sqrt{2}}{2} \cdot \left(-\frac{3}{\sqrt{10}}\right)}{\sqrt{2} \cdot \left(-\frac{8}{10}\right) - \frac{\sqrt{2}}{2} \cdot \left(-\frac{3}{\sqrt{10}}\right)}$$

$$= \frac{\frac{9}{10} - \frac{3}{\sqrt{10}}}{\frac{8}{10} - \frac{3}{\sqrt{10}}} = \frac{-9 - 3\sqrt{10}}{8 - 3\sqrt{10}}$$

$$= -\frac{3(3 + \sqrt{10})}{3(\sqrt{10} - 3)}$$

B)

2/урд 95.

$$\text{мы } \sin(\alpha + \beta) = 0. \text{ так как } \alpha, \beta \in [0, \pi] \text{ тогда } \sin \alpha = \frac{4}{5}; \cos \beta = -\frac{3}{5}$$

$$\frac{\pi}{2} < \alpha < \pi \Rightarrow \alpha \in [0, \pi] \text{ так как } \alpha \in [0, \pi] \text{ тогда } \frac{\pi}{2} < \beta < \pi \Rightarrow \beta \in [0, \pi] \text{ так как } \beta \in [0, \pi]$$

$$\text{тогда } \sin \alpha = \frac{4}{5}, \text{ так как } \cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

$$\text{тогда } \cos \beta = -\frac{3}{5}, \text{ так как } \sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{4}{5} \cdot \left(-\frac{3}{5}\right) + \left(\frac{3}{5}\right) \cdot \frac{4}{5} = -\frac{12}{25} + \frac{12}{25} = 0$$

В) $\sin(\alpha - \beta) = 0$, так как

$$\sin \alpha = \frac{24}{25}, \cos \beta = -\frac{4}{5}; 0 < \alpha < \frac{\pi}{2} \Rightarrow \alpha \in [0, \frac{\pi}{2}] \text{ так как } \alpha \in [0, \frac{\pi}{2}] \text{ тогда } \frac{\pi}{2} < \beta < \pi$$

$\beta \in [0, \pi]$ так как

$$\sin \alpha = \frac{24}{25} \Rightarrow \cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{576}{625}} = \frac{7}{25}$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta = \frac{24}{25} \cdot \left(-\frac{4}{5}\right) - \frac{7}{25} \cdot \frac{3}{5} = -\frac{96}{125} - \frac{21}{125} = -\frac{117}{125}$$

$$= -\frac{96}{125} - \frac{21}{125} = -\frac{117}{125}$$

9) $\cos(\alpha + \beta)$ - 6 tpt

$$\cos \alpha = \frac{5}{13}; \sin \beta = \frac{12}{13}; 0 < \alpha < \frac{\pi}{2}, \frac{\pi}{2} < \beta < \pi$$

$$(\cos \alpha = \sqrt{1 - \sin^2 \alpha})$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

$$\cos \beta = -\sqrt{1 - \sin^2 \beta} = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{5}{13} \cdot \left(-\frac{5}{13}\right) - \frac{12}{13} \cdot \frac{12}{13}$$

$$= \frac{-25 - 144}{169} = -\frac{169}{169} = -1$$

2/10/2019 96

10/10/2019 $\sin(\alpha + \beta)$ 6 tpt

$$\sin \alpha = \frac{3}{5}; \sin \beta = \frac{7}{25}; \frac{\pi}{2} < \alpha < \pi \quad \cos(\alpha - \beta) > 0$$

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5}$$

$$\cos \beta = \sqrt{1 - \sin^2 \beta} = \sqrt{1 - \frac{49}{625}} = \frac{24}{25}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta =$$

$$= -\frac{4}{5} \cdot \frac{24}{25} + \frac{3}{5} \cdot \frac{7}{25} = -\frac{96 + 21}{125} = -\frac{117}{125} < 0$$

$$\cos(\alpha - \beta) = -\frac{4}{5} \left(-\frac{24}{5}\right) + \frac{3}{5} \cdot \frac{7}{25} = \frac{96 + 21}{125} = \frac{117}{125} > 0$$

$$\Rightarrow \cos \beta = -\frac{24}{25}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{3}{5} \cdot \left(-\frac{24}{25}\right) +$$

$$+ \left(-\frac{4}{5}\right) \cdot \frac{7}{25} = -\frac{72}{125} - \frac{28}{125} = -\frac{100}{125}$$

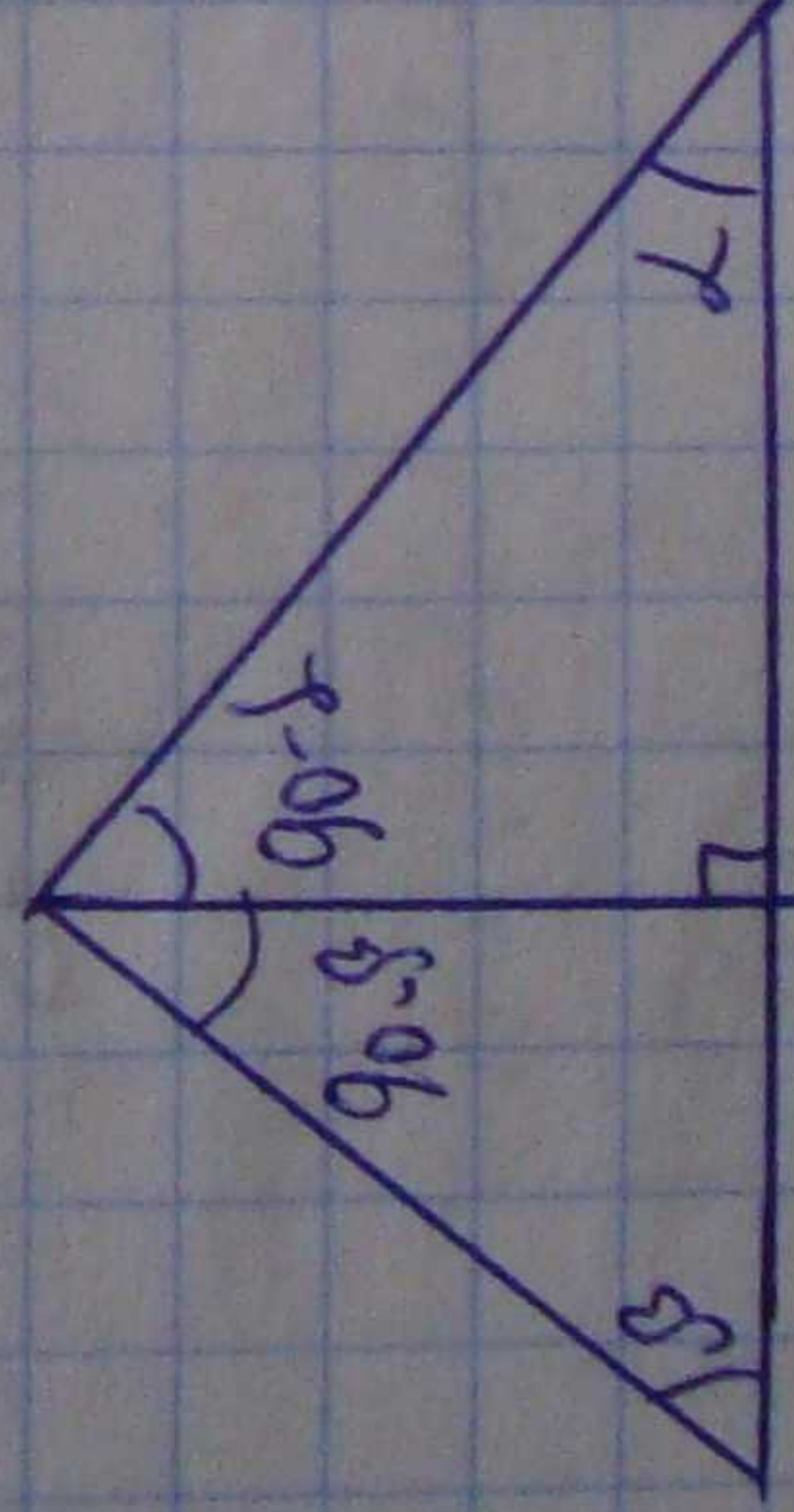
$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta =$$

$$= -\frac{4}{5} \cdot \left(-\frac{24}{25}\right) - \frac{3}{5} \cdot \frac{7}{25} = \frac{96}{125} - \frac{21}{125} = \frac{75}{125}$$

$$P) \sin \alpha = \frac{3}{5}; \cos \beta = \frac{3}{5}; \frac{\pi}{2} < \alpha < \pi \text{ u } \cos(\alpha - \beta) < 0$$

Answer 94.

$$\cos \alpha = \frac{1}{\sqrt{5}}; \cos \beta = \frac{1}{\sqrt{10}}$$



$$\cos \alpha = \frac{1}{\sqrt{5}} \Rightarrow \sin(90 - \alpha) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos(90 - \alpha) = \sqrt{1 - \sin^2(90 - \alpha)} = \sqrt{1 - \frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \beta = \frac{1}{\sqrt{10}} \Rightarrow \sin(90 - \beta) = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos(90 - \beta) = \sqrt{1 - \sin^2(90 - \beta)} = \sqrt{1 - \frac{1}{10}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{1}{10}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos((90 - \alpha) + (90 - \beta)) = \cos(90 - \alpha) \cos(90 - \beta) - \sin(90 - \alpha) \sin(90 - \beta) =$$

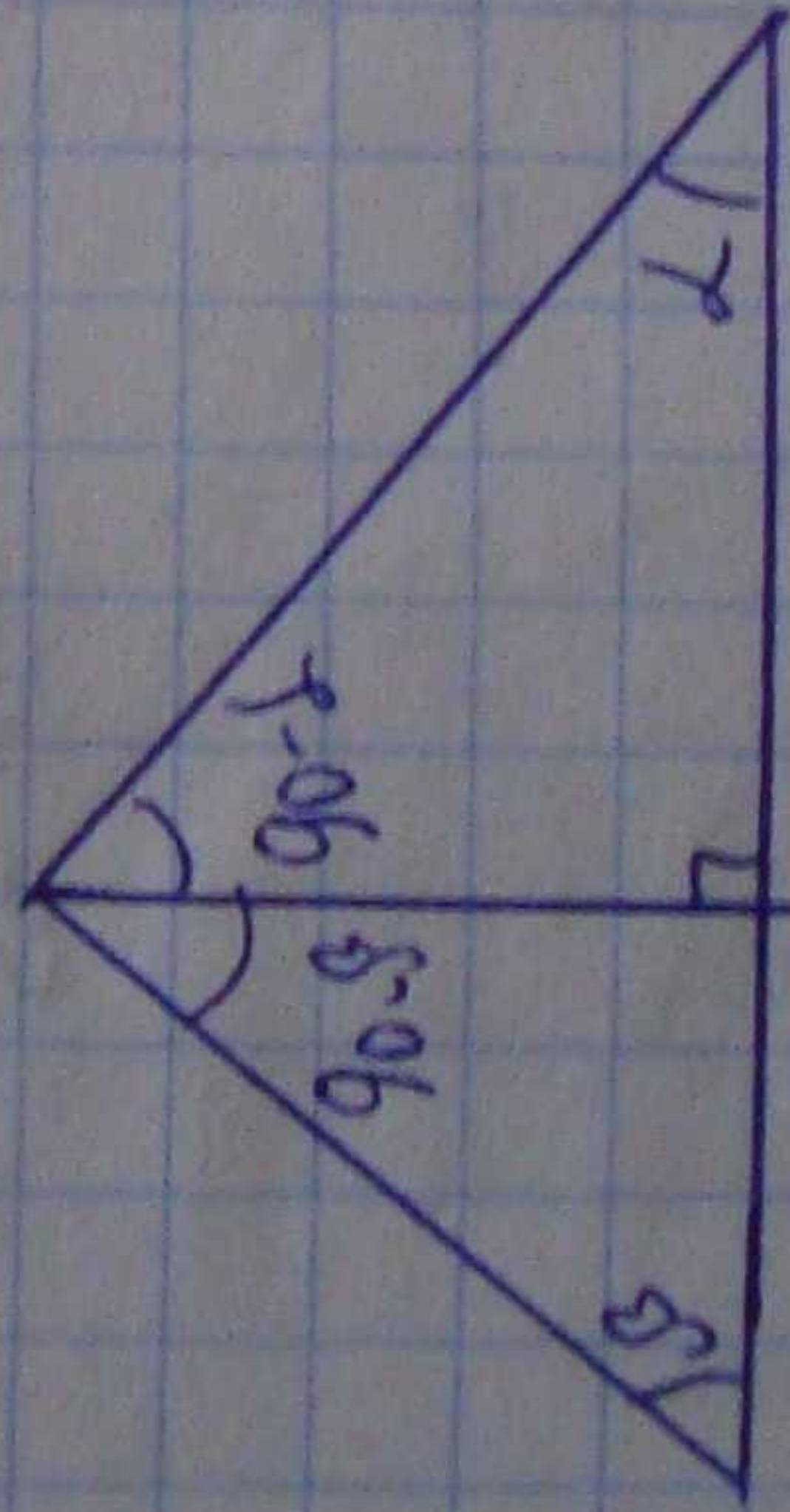
$$\Rightarrow \cos \beta = -\frac{24}{25}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{3}{5} \cdot \left(-\frac{24}{25}\right) + \left(-\frac{4}{5}\right) \cdot \frac{7}{25} = -\frac{72}{125} - \frac{28}{125} = -\frac{100}{125}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = -\frac{4}{5} \cdot \left(-\frac{24}{25}\right) - \frac{3}{5} \cdot \frac{7}{25} = \frac{96}{125} - \frac{21}{125} = \frac{75}{125}$$

$$A) \sin \alpha = \frac{3}{5}; \cos \beta = \frac{3}{5}; \frac{\pi}{2} < \alpha < \pi; \text{ b) } \cos(\alpha - \beta) < 0$$

aufwand 94.



$$\cos \alpha = \frac{1}{\sqrt{5}}; \cos \beta = \frac{1}{\sqrt{10}}$$

$$\cos \alpha = \frac{1}{\sqrt{5}} \Rightarrow \sin(90 - \alpha) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos(90 - \alpha) = \sqrt{1 - \sin^2(90 - \alpha)} = \sqrt{1 - \frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \beta = \frac{1}{\sqrt{10}} \Rightarrow \sin(90 - \beta) = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos(90 - \beta) = \sqrt{1 - \sin^2(90 - \beta)} = \sqrt{1 - \frac{1}{10}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{1}{10}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos((90 - \alpha) + (90 - \beta)) = \cos(90 - \alpha) \cos(90 - \beta) - \sin(90 - \alpha) \sin(90 - \beta) =$$

$$= \frac{2\sqrt{5}}{5}, \frac{3\sqrt{10}}{10} - \frac{\sqrt{5}}{5} \cdot \frac{\sqrt{10}}{10} \cdot \frac{6\sqrt{50} - \sqrt{50}}{50} \cdot \frac{\sqrt{50} \cdot 8}{50} \cdot \frac{\sqrt{50}}{10} =$$

$$= \frac{\sqrt{50}}{\sqrt{10}^2} \cdot \sqrt{\frac{50}{100}} \cdot \frac{1}{2}$$

2/4/2018

$$u) \left(\frac{5(m-2)}{m^3-8} - \frac{m+2}{m^2+2m+4} \right) \cdot \frac{2m^2+4m+8}{m-3} \cdot z$$

$$= \left(\frac{5(m-2)}{(m-2)(m^2+2m+4)} - \frac{m+2}{m^2+2m+4} \right) \cdot \frac{2(m^2+2m+4)}{m-3} \cdot z$$

$$= \left(\frac{5(m-2) - (m+2)(m-2)}{(m-2)(m^2+2m+4)} \right) \cdot \frac{2(m^2+2m+4)}{m-3} \cdot z$$

$$= \frac{(m-2)(5-m-2)}{m-2} \cdot \frac{2}{m-3} \cdot z$$

$$= \frac{-2(m-3)}{m-3} \cdot z = -2 \cdot z$$

$$A) \left(\frac{n+2}{3n} - \frac{2}{n-2} - \frac{n-14}{3n^2-6n} \right) \cdot \frac{n+2}{6n} \cdot \frac{1}{n-5} \cdot z$$

$$= \left(\frac{n+2}{3n} - \frac{2}{n-2} - \frac{n-14}{3n(n-2)} \right) \cdot \frac{n+2}{6n} \cdot \frac{1}{n-5} \cdot z$$

$$= \left(\frac{(n-2)(n+2)-6n-(n-14)}{3n(n-2)} \right) \cdot \frac{6n^2}{n+2} \cdot \frac{1}{n-5} \cdot z$$

$$z \frac{n^2 - 4 - 6n - n + 14}{n^2 - 3n - 10} \cdot z \frac{n^2 - 7n + 10}{(n+2)(n-5)} \cdot z \frac{(n-2)(n-5)}{(n+2)(n-5)} \cdot z$$

$$n^2 + 7n + 10 = 0$$

$$20 = 49 - 40 = 9$$

$$n_{1,2} = \frac{7 \pm 3}{2} = 5$$

2/урд 102

$$a = 6$$

$$w) \left(\frac{a\sqrt{a}-1}{\sqrt{a}+1} : (a-1) + \frac{\sqrt{a}}{(\sqrt{a}+1)^2} \right) (2a+1) z$$

$$z \left(\frac{(\sqrt{a}-1)(a+\sqrt{a}+1)}{\sqrt{a}+1} \cdot \frac{1}{(\sqrt{a}-1)(\sqrt{a}+1)} + \frac{\sqrt{a}}{(\sqrt{a}+1)^2} \right) (2a+1) z$$

$$z \frac{a+\sqrt{a}+1+\sqrt{a}}{(\sqrt{a}+1)^2} \cdot (2a+1) z = \frac{(\sqrt{a}+1)^2}{(\sqrt{a}+1)^2} \cdot 2a+1 = 13$$

$$1750:13$$

2/урд 103

$$w) \frac{\sin 2L}{\sin L} z \frac{2 \sin L \cos L}{\sin L} z 2 \cos L$$

$$p) \frac{\sin 2L}{2 \cos^2 L} z \frac{2 \cos L \sin L}{\cos 2L + 1} z \frac{\cos L \sin L}{\cos 2L + \sin^2 L} z \frac{\cos L \sin L}{1 - 2 \sin^2 L + \sin^2 L} z$$

$$f) \frac{\sin 2\alpha}{2\cos^2 \alpha} = \frac{2\sin \alpha \cos \alpha}{2\cos^2 \alpha}, \quad \frac{\sin \alpha}{\cos \alpha} = \operatorname{tg} \alpha$$

$$g) \frac{\sin 2\alpha}{\sin \alpha} = \frac{2\cos \alpha \sin \alpha}{\sin \alpha} = \cos \alpha$$

$$h) \cos 2\alpha - \cos^2 \alpha = 2\cos^2 \alpha - 1 - \cos^2 \alpha = \cos^2 \alpha - 1 = -\sin^2 \alpha$$

$$b) \sin^2 \alpha + \cos 2\alpha = \sin^2 \alpha + 1 - 2\sin^2 \alpha = 1 - \sin^2 \alpha = \cos^2 \alpha$$

$$y) \sin^2 \alpha - \cos^2 \alpha = -\cos 2\alpha$$

2/4/2/104

$$\text{мы } 2 \sin \frac{\pi}{12} \cos \frac{\pi}{12} = \sin 2 \cdot \frac{\pi}{12} = \sin \frac{\pi}{6} = \frac{1}{2}$$

$$f) \cos^2 15^\circ - \sin^2 15^\circ = \cos 2 \cdot 15^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$g) 8 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} = 2 \sin^2 2 \cdot \frac{\pi}{8} = 2 \sin^2 \frac{\pi}{4} = 1$$

$$h) 2 \left(\sin^2 \frac{\pi}{8} - \cos^2 \frac{\pi}{8} \right)^2 = 2 \left(-\cos \frac{\pi}{4} \right)^2 = 2 \cdot \cos^2 \frac{\pi}{4} = 1$$

$$b) 4 \sin 105^\circ \cos 105^\circ = 2 \cdot \sin 2 \cdot 105^\circ = 2 \cdot \sin 210^\circ = -1$$

$$y) \cos^2 165^\circ - \sin^2 165^\circ = \cos 2 \cdot 165^\circ = \cos 330^\circ = \frac{\sqrt{3}}{2}$$

2/4/2/105

$$\text{мы } \frac{2 \operatorname{tg} 15^\circ}{1 - \operatorname{tg}^2 15^\circ} = \operatorname{tg} 2 \cdot 15^\circ = \operatorname{tg} 30^\circ = \frac{\sqrt{3}}{3}$$

$$p) \frac{2\sqrt{3} \operatorname{tg} 75^\circ}{1 - \operatorname{tg}^2 75^\circ} = \sqrt{3} \operatorname{tg} 2 \cdot 75^\circ = \sqrt{3} \operatorname{tg} 150^\circ = 1$$

$$a) \frac{\operatorname{ctg}^2 \frac{3\pi}{8} - 1}{2 \operatorname{ctg} \frac{3\pi}{8}} = \operatorname{ctg} \frac{3\pi}{4} = -1$$

$$b) \frac{4 \operatorname{tg} \frac{\pi}{8}}{1 - \operatorname{tg}^2 \frac{\pi}{8}} = 2 \operatorname{tg} \frac{\pi}{4} = 2$$

$$c) \frac{1 - \operatorname{tg}^2 45^\circ}{2 \operatorname{tg} 45^\circ} = \operatorname{ctg} 2 \cdot 45^\circ = \operatorname{ctg} 90^\circ = -\sqrt{3}$$

$$d) \frac{2\sqrt{3} \operatorname{ctg} \frac{\pi}{8}}{\operatorname{ctg}^2 \frac{\pi}{8} - 1} = \sqrt{3} \operatorname{tg} \frac{\pi}{8} = \sqrt{3} \cdot 1 = \sqrt{3}$$

з/урд 106

$$u) \sin \frac{\pi - \alpha}{2} = \cos \frac{\pi - \alpha}{2} = \frac{\sin \alpha}{2}$$

$$p) \frac{2 \cos^2 \frac{\pi + \alpha}{4} - 2 \sin^2 \frac{\pi + \alpha}{4}}{4} = 2 \left(\cos \frac{\pi + \alpha}{2} \right) = 2 \cos \left(\frac{\pi}{2} + \frac{\alpha}{2} \right) =$$

$$= -2 \sin \frac{\alpha}{2}$$

$$q) \frac{4 \operatorname{tg} \frac{3\pi - \alpha}{2}}{1 - \operatorname{tg}^2 \frac{3\pi - \alpha}{2}} = 2 \operatorname{tg}(3\pi - \alpha) = 2 \operatorname{tg}(\pi - \alpha) = -2 \operatorname{tg} \alpha$$

$$r) 4 \sin \frac{\alpha}{2} \sin \frac{\pi - \alpha}{2} = 4 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} =$$

$$= -2 \sin \alpha \cos \alpha = -\sin 2\alpha$$

г)

2/uyrd 107.

$$\text{uf } \frac{\sin 2x - 2\sin x}{\cos x - 1} = \frac{2\sin x \cos x - 2\sin x}{\cos x - 1} = 2\sin x$$

$$p) \sin 2x \cot x - 1 = 2\sin x \cos x \cdot \frac{\cos x}{\sin x} - 1 = 2\cos^2 x - 1 = \cos 2x$$

$$q) \frac{\cos 2x - \cos^2 x}{1 - \cos^2 x} = \frac{2\cos^2 x - 1 - \cos^2 x}{1 - \cos^2 x} = \frac{\cos^2 x - 1}{-(\cos^2 x - 1)} = -1$$

1

2/uyrd 108

$$\text{uf } 1 + \sin 2x = (\sin x + \cos x)^2$$

$$1 + \sin 2x = 1 + 2\sin x \cos x = \sin^2 x + \cos^2 x + 2\sin x \cos x = (\sin x + \cos x)^2$$

$$p) 1 - \sin 2x = (\sin x - \cos x)^2$$

$$1 - \sin 2x = \sin^2 x + \cos^2 x - 2\sin x \cos x = (\sin x - \cos x)^2$$

$$q) \frac{\cos 2x}{1 + \sin 2x} = \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$\frac{\cos 2x}{1 + \sin 2x} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} =$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$9) \frac{1 - \cos x + \cos 2x}{\sin 2x - \sin x} = \operatorname{ctg} x$$

$$\frac{1 - \cos x + \cos^2 x - \sin^2 x}{2 \sin x \cos x - \sin x} = \frac{\cos x (2 \cos x - 1)}{\sin x (2 \cos x - 1)} = \operatorname{ctg} x$$

зупыд 109.

$$\text{мы } \operatorname{tg} 2x - \operatorname{tg} x = \frac{\operatorname{tg} x}{\cos 2x}$$

$$\operatorname{tg} 2x - \operatorname{tg} x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} - \operatorname{tg} x =$$

$$= \frac{2 \operatorname{tg} x}{\cos^2 x - \sin^2 x} \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{2 \operatorname{tg} x \cdot \cos^2 x}{\cos^2 x} - \operatorname{tg} x =$$

$$= \frac{2 \operatorname{tg} x \cos^2 x}{\cos 2x} - \frac{\sin x}{\cos x} =$$

$$= \frac{\operatorname{tg} x}{\cos 2x} \cdot (\cos 2x + 1) - \frac{\sin x}{\cos x} = \frac{\operatorname{tg} x}{\cos 2x} \cdot$$

$$= \operatorname{tg} x \left(\frac{\operatorname{tg} x}{1 - \operatorname{tg}^2 x} - 1 \right) = \operatorname{tg} x \left(\frac{\operatorname{tg} x - 1 + \operatorname{tg}^2 x}{1 - \operatorname{tg}^2 x} \right) =$$

$$\frac{-\sin x}{x}$$

$$= \operatorname{tg} x \left(\frac{\operatorname{tg} x}{\cos 2x} - 1 \right) = \operatorname{tg} x \left(\frac{\operatorname{tg} x}{\cos 2x} - \frac{\cos^2 x - \sin^2 x}{\cos 2x} \right)$$

$$= \operatorname{tg} x \left(\frac{\operatorname{tg} x}{\cos 2x} \right)$$

2/11/21 110

$$\text{w/ } \frac{1 + \sin 2x}{(\sin x + \cos x)^2} = \frac{1 + \sin 2x}{1 + \sin 2x} = 1$$

$$2) 1 - 2 \sin^2 \frac{x}{8} = \sin^2 \frac{x}{8} + \cos^2 \frac{x}{8} - 2 \sin^2 \frac{x}{8} =$$

$$= \cos^2 \frac{x}{8} - \sin^2 \frac{x}{8} = \cos \frac{x}{4}$$

$$3) 2 \cos^2 \frac{x}{6} - 1 = 2 \cos^2 \frac{x}{6} - \cos^2 \frac{x}{6} - \sin^2 \frac{x}{6} =$$

$$= \cos^2 \frac{x}{6} - \sin^2 \frac{x}{6} = \cos \frac{x}{3}$$

$$4) 1 - 8 \cos^2 x \sin^2 x = 1 - 4 \sin 2x$$

$$= (\sin^2 x - \cos^2 x)^2 - 6 \cos^2 x \sin^2 x = (\sin x - \cos x)^2 - 3 \sin 2x$$

2/11/21 111

$$\text{w/ } 2 \cos^2 x - \cos 2x = \cos 2x + 1 - \cos 2x = 1$$

$$1) \frac{2 \sin x + \sin 2x}{2 \sin x + \sin 2x} = \frac{2 \sin x - 2 \sin x \cos x}{2 \sin x + 2 \sin x \cos x} = \frac{1 - \cos x}{1 + \cos x}$$

$$2) \frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \frac{2 \sin^2 x + 2 \sin x \cos x}{2 \cos^2 x + 2 \sin x \cos x} =$$

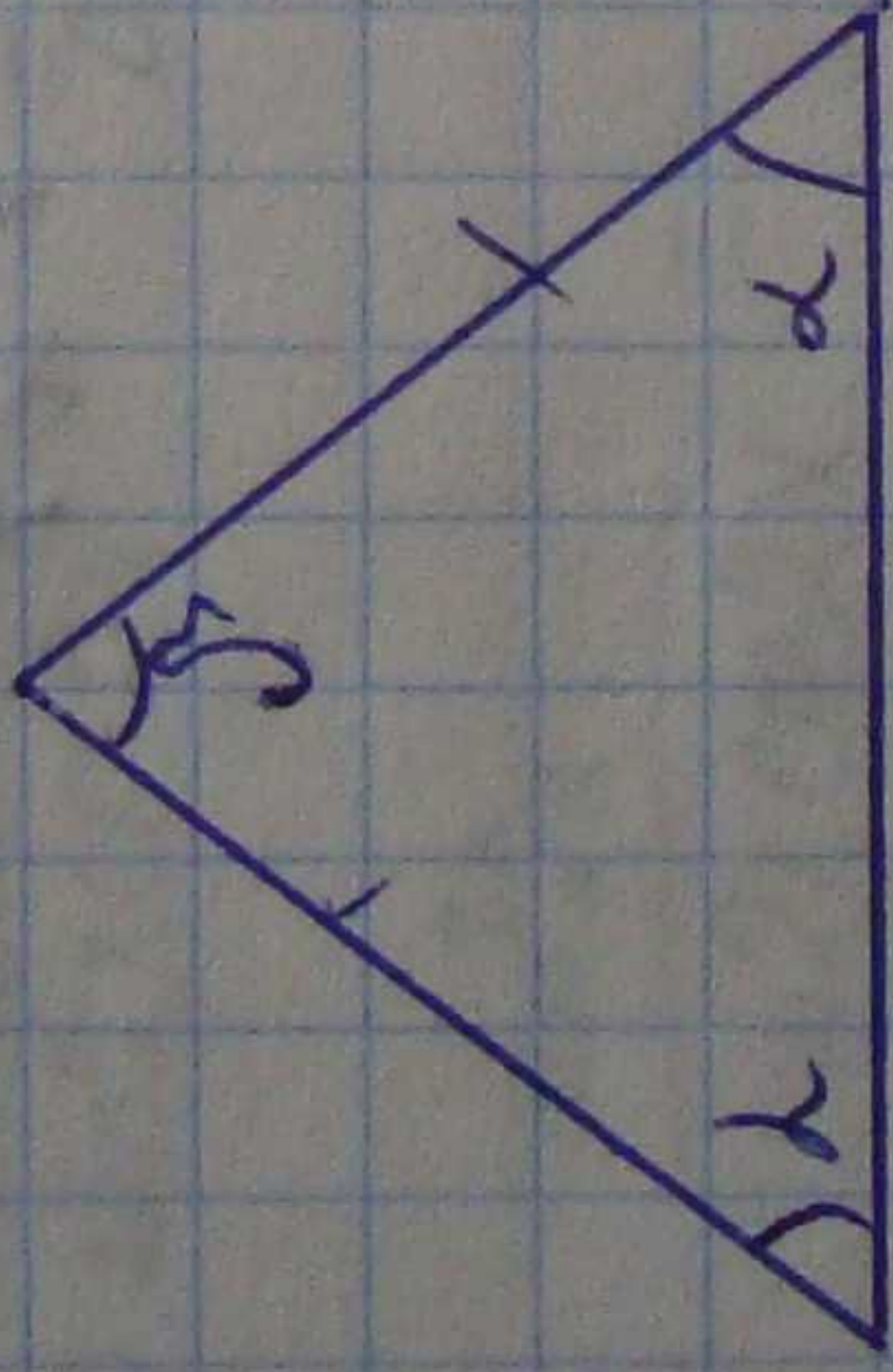
$$= \frac{2 \sin x (\sin x + \cos x)}{2 \cos x (\cos x + \sin x)} = \tan x$$

$$\begin{aligned}
 \eta) & \frac{\cos^2 2\alpha - 4\cos^2 \alpha + 3}{\cos^2 2\alpha + 4\cos^2 \alpha - 1} = \frac{(2\cos^2 \alpha - 1)^2 - 4\cos^2 \alpha + 3}{(2\cos^2 \alpha - 1)^2 + 4\cos^2 \alpha - 1} \\
 & = \frac{4\cos^4 \alpha - 4\cos^2 \alpha + 1 - 4\cos^2 \alpha + 3}{4\cos^4 \alpha - 4\cos^2 \alpha + 1 + 4\cos^2 \alpha - 1} = \frac{4(\cos^4 \alpha - 2\cos^2 \alpha + 1)}{4\cos^4 \alpha} \\
 & = \frac{(\cos^2 \alpha + 1)^2}{\cos^4 \alpha} \cdot \frac{(\cos^2 \alpha - \cos^2 \alpha - \sin^2 \alpha)^2}{\cos^4 \alpha} = \frac{\sin^4 \alpha}{\cos^4 \alpha} = \operatorname{tg}^4 \alpha.
 \end{aligned}$$

112/

$$\cos \alpha = 0,6$$

$$\sin \beta \text{ in } \cos \beta = ?$$



$$\begin{aligned}
 \cos \beta & = \cos (180 - 2\alpha) = \cos (2(90 - \alpha)) = \sin 2\alpha = \\
 & = 2 \sin \alpha \cos \alpha
 \end{aligned}$$

finden wir $\cos \alpha = 0,6$, wegen $\sin \alpha = \sqrt{1 - 0,36} = 0,8$

$$\cos \beta = 2 \sin \alpha \cos \alpha = 2 \cdot 0,6 \cdot 0,8 = 0,96 = \frac{24}{25}$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \left(\frac{24}{25}\right)^2} = \frac{7}{25}$$

zum 113

$$\begin{aligned}
 \text{wir } \frac{2}{\sin 4x} - \operatorname{ctg} 2x & = \operatorname{tg} 2x \\
 \frac{2}{\sin 4x} - \operatorname{ctg} 2x & = \frac{2}{2 \sin 2x \cos 2x} - \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x} =
 \end{aligned}$$

$$z = \frac{2}{4 \sin x \cos x (\cos^2 x - 1)}$$

Задача 114

Решение
 Пусть $25 \sin 2\alpha = 6$, тогда $\cos \alpha = -0,6$ $6 \quad \pi < \alpha < \frac{3}{2}\pi$

$$(25 \sin 2\alpha \cdot 225 \sin \alpha \cos \alpha =)$$

$$\cos \alpha = -0,6 \Rightarrow -\frac{3}{5} \quad ; \quad \sin \alpha = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$25 \sin 2\alpha = 50 \sin \alpha \cos \alpha = 50 \cdot \frac{3}{5} \cdot \frac{4}{5} = 24$$

$$p) \quad 18 \cos 2\alpha = 6, \quad \text{тогда} \quad \cos \alpha = -\frac{\sqrt{11}}{6}$$

$$18 \cos 2\alpha = 18 \cdot (2 \cos^2 \alpha - 1) =$$

$$= 18 \cdot \left(\frac{11}{18} - 1 \right) = 18 \cdot \left(-\frac{7}{18} \right) = -7$$

$$q) \quad \sqrt{2} \operatorname{tg} 2\alpha = 6, \quad \text{тогда} \quad \cos \alpha = \frac{\sqrt{37}}{3} \quad 4 \quad \frac{7}{2}\pi < \alpha < 4\pi$$

$$\times \quad \sin \alpha = -\sqrt{1 - \frac{37}{9}} = -\sqrt{\frac{6}{9}} = -\frac{\sqrt{6}}{3}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = -\frac{\sqrt{6}}{3} \cdot \frac{3}{\sqrt{37}} = -\frac{\sqrt{2}}{\sqrt{37}}$$

$$\sqrt{2} \operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} = \frac{-4}{1 - 2} = 4$$

Задача 113

$$\text{в) } \frac{2}{\sin 4x} - \operatorname{ctg} 2x = \operatorname{tg} 2x$$

$$\frac{2}{2 \sin 2x \cos 2x} - \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x} = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$

$$\frac{2}{2 \sin 2x \cos 2x} - \frac{\cos 2x}{\sin 2x} = \frac{\sin 2x}{\cos 2x}$$

$$\frac{2}{2 \sin 2x \cos 2x} - \frac{\cos 2x}{\sin 2x} = \frac{2 - 2 \cos^2 2x}{2 \sin 2x \cos 2x} = \frac{1 - \cos^2 2x}{\sin 2x \cos 2x}$$

$$= \frac{\sin^2 2x}{\sin 2x \cos 2x} = \frac{\sin 2x}{\cos 2x} = \operatorname{tg} 2x$$

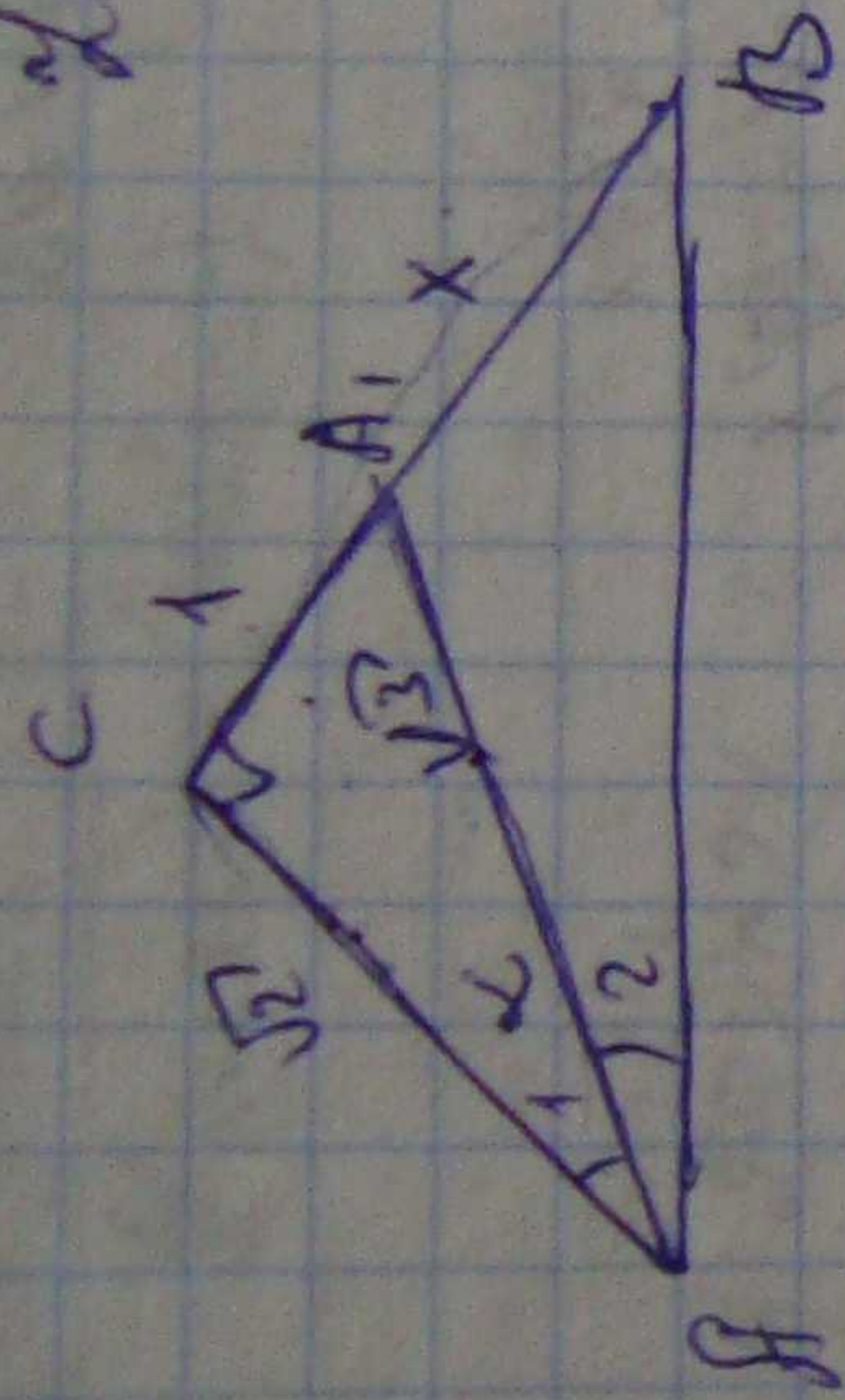
$$\text{р) } \frac{\sin 4x}{1 + \cos 4x} \cdot \frac{\cos 2x}{1 + \cos 2x} = \operatorname{tg} x$$

$$\frac{2 \sin 2x \cos 2x}{2 \cos^2 2x} \cdot \frac{\cos 2x}{2 \cos^2 x} = \frac{\sin 2x \cdot 2 \sin x \cos x}{2 \cos^2 x \cdot 2 \cos^2 x} = \frac{\sin x}{\cos x} = \operatorname{tg} x$$

$$\text{г) } \sin 4x + \cos 4x \operatorname{ctg} 2x = \frac{1 - \operatorname{tg}^2 x}{2 \operatorname{tg} x}$$

$$\sin 4x + \frac{\cos 4x \cdot \cos 2x}{\sin 2x} = \frac{\sin 4x \sin 2x + \cos 4x \cos 2x}{\sin 2x} = \frac{1}{\operatorname{tg} 2x} = \frac{1}{\frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}} = \frac{1 - \operatorname{tg}^2 x}{2 \operatorname{tg} x}$$

Ex 116



$$AB = \sqrt{3}$$

$$AC = \sqrt{2}$$

$$CB = ?$$

$$AB = x$$

$$CA' = 1$$

$$\angle 1 = \angle 2$$

$$\frac{AC}{AB} = \frac{1}{x}$$

$$AB = x\sqrt{2}$$

$$(x\sqrt{2})^2 = 2 + (1+x)^2$$

$$2x^2 = 2 + 1 + 2x + x^2$$

$$x^2 - 2x - 3 = 0$$

$$x = 1 \pm 2 = 3$$

$$\angle 1 = \angle 2 \Rightarrow \angle A = 2\alpha$$

$$\tan 2\alpha = \frac{CB}{AC}$$

$$CB = \sqrt{2} \tan 2\alpha$$

$$\Delta CA'B' \text{ by } \tan \alpha = \frac{CA'}{CB'} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}$$

$$CB = \sqrt{2} \tan 2\alpha = \sqrt{2} \cdot \frac{2 \tan \alpha}{1 - \tan^2 \alpha} =$$

$$= \frac{2\sqrt{2} \cdot \frac{1}{\sqrt{2}}}{1 - \frac{1}{2}} = 2 \cdot \frac{2}{1} = 4$$

2/4/24 124

$$9) \frac{1 + \cos 4x}{\cos^2 x - \sin^2 x} = \frac{1 + \cos^2 2x + \sin^2 2x}{\cos 2x} = \frac{2\cos^2 2x}{\cos 2x}$$

$$7) \frac{1 - \cos 3x}{1 + \cos 3x} = \frac{2\sin^2 \frac{3}{2}x}{2\cos^2 \frac{3}{2}x} = \tan^2 \frac{3}{2}x$$

2/4/25 125

$$F) 1 - \sin x = 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right) \quad | 1 - \cos x = 2\sin^2 \frac{x}{2}$$

$$1 - \sin x = 1 - \cos\left(\frac{\pi}{2} - x\right) = 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

Thema: trigonometrische Funktionen

$$\sin^2 \beta = \frac{1 - \cos 2\beta}{2}$$

$$2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \frac{2\left(1 - \cos 2\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)}{2}$$

$$= 1 - \cos\left(\frac{\pi}{2} - x\right) = 1 - \sin x$$

$$AB = BC = 9$$

$$AC = 14$$

$$BE = ?$$



9)

Задача 130

а) $\sqrt{118} \sin \frac{\alpha}{2}$, $6 \text{ pt } \lg \alpha = -\frac{24\sqrt{6}}{5}$ $\wedge \frac{5}{2}\pi < \alpha < 3\pi$.

$$\cos^2 \alpha = \frac{1}{1 + \lg^2 \alpha}$$

$$\frac{\pi}{2} < \alpha < \pi$$

$$= \frac{1}{1 + \frac{24^2 \cdot 6}{25}} = \frac{1}{25 + 3456} = \frac{1}{3481}$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$= \frac{25}{3481}$$

$$\frac{109}{9} \sqrt{\frac{3481}{25}} = 59$$

$$\cos \alpha = \pm \frac{5}{59}$$

$$\cos \alpha = -\frac{5}{59}$$

$$= \sqrt{118} \sin \frac{\alpha}{2} = \sqrt{1 + \frac{5}{59}} = \sqrt{\frac{64}{59} \cdot \frac{1}{2}}$$

$$= \sqrt{\frac{32}{59}} \cdot \sqrt{118} = \sqrt{\frac{32}{59} \cdot 118} = \sqrt{64} = 8$$

Задача 131

а) $\sqrt{2} \lg \frac{\alpha}{2}$; $\left| \cos \alpha - \frac{1}{6} \right| = \frac{1}{2}$ $\wedge \pi < \alpha < \frac{3}{2}\pi$

$$\left| \cos \alpha - \frac{1}{6} \right| = \frac{1}{2}$$

$$\cos \alpha - \frac{1}{6} = 0$$

$$\cos \alpha = \frac{1}{6}$$

$$\cos \alpha = \frac{1}{6}$$

$$\cos \alpha - \frac{1}{6} = \frac{1}{2}$$

$$\cos \alpha = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$$

$$\cos \alpha \in \left[\frac{1}{6}, +\infty \right)$$

$$\cos \alpha = \frac{2}{3}$$

$$\cos \alpha \in \left(-\infty, \frac{1}{6} \right]$$

$$\cos \alpha = -\frac{1}{3}$$

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2} \left(1 \right) = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\sqrt{2} \tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \sqrt{\frac{1 + \frac{1}{3}}{1 - \frac{1}{3}}} = \sqrt{\frac{4}{3} \cdot \frac{3}{2}} = \sqrt{2}$$

$$\frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{4}$$

$$\sqrt{2} \tan \frac{\alpha}{2} = -\sqrt{2} \cdot \sqrt{2} = -2$$

since

$$\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\sin \alpha \cos \beta = \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta)) \quad (1)$$

$$\frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta)) \quad (2)$$

$$\sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta)) \quad (3)$$

$$\sin \alpha \cos \beta = \frac{1}{2} \left(\sin \left(\frac{\alpha + \beta}{2} \right) + \sin \left(\frac{\alpha - \beta}{2} \right) \right)$$

$$\begin{cases} \alpha + \beta = x \\ \alpha - \beta = y \end{cases} \quad \left| \begin{array}{l} \alpha = \frac{x+y}{2} \\ \beta = \frac{x-y}{2} \end{array} \right.$$

(1) formula supplied.

$$\sin x + \sin y = 2 \sin x \cos \beta = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \quad (4)$$

(2)

$$\cos x + \cos y = \cos \frac{x+y}{2} \cos \frac{x-y}{2} \quad (5)$$

(3) - mod

$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{x-y}{2} \quad (6)$$

(4) - for Stg y - c. for h. - y - n. l.

$$(7) \sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$\lg x \pm \lg y = \frac{\sin x}{\cos x} \pm \frac{\sin y}{\cos y} =$$

$$= \frac{\sin x \cos y \pm \sin y \cos x}{\cos x \cos y} = \frac{\sin(x \pm y)}{\cos x \cos y} \quad (8)$$

Math. c.

137 - 141

22.02.06p.

2/4/2012

$$\text{w/ } \alpha = 22.5^\circ$$

$$\sin 22.5^\circ = \sin \frac{45^\circ}{2} = \frac{1}{2} \sqrt{1 - \cos 45^\circ} = \frac{1}{2} \sqrt{1 - \frac{\sqrt{2}}{2}} = \frac{1}{2} \sqrt{\frac{2 - \sqrt{2}}{2}}$$

$$z = \frac{\sqrt{2} - 1}{2} + i \frac{\sqrt{2}(\sqrt{2} - 1)}{2}$$

$$\cos 22.5^\circ = \cos \frac{45^\circ}{2} = \frac{1}{2} \sqrt{1 + \frac{\sqrt{2}}{2}} = \frac{1}{2} \sqrt{\frac{2 + \sqrt{2}}{2}}$$

$$\lg 22.5^\circ = \lg \frac{45^\circ}{2} = \frac{1}{2} \sqrt{\frac{1 - \cos 45^\circ}{1 + \cos 45^\circ}} = \frac{1}{2} \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}}$$

$$z = \frac{1}{2} \sqrt{(2 - \sqrt{2})^2} = \frac{1}{2} \sqrt{2 - \sqrt{2}} - i \frac{1}{2} \sqrt{2 - \sqrt{2}}$$

$$\lg 22.5^\circ = \frac{1}{2} \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}} = \frac{1}{2} \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}}$$

$$P) \alpha = \frac{3\pi}{8}$$

$$\sin \frac{3\pi}{8} = \sin \frac{1}{2} \cdot \frac{3\pi}{4} = \frac{1}{2} \sqrt{1 - \cos \frac{3\pi}{4}} = \frac{1}{2} \sqrt{1 - \frac{-\sqrt{2}}{2}} = \frac{1}{2} \sqrt{\frac{2 + \sqrt{2}}{2}}$$

$$z = \frac{\sqrt{2}(\sqrt{2} + 1)}{2}$$

$$\cos \frac{3\pi}{8} = \cos \frac{1}{2} \cdot \frac{3\pi}{4} = \frac{1}{2} \sqrt{1 + \cos \frac{3\pi}{4}} = \frac{1}{2} \sqrt{1 + \frac{-\sqrt{2}}{2}} = \frac{1}{2} \sqrt{\frac{2 - \sqrt{2}}{2}}$$

2/4/2012 12:3

$$\text{uf) } 1 + \cos 4x = 2 \cos^2 2x$$

$$\text{P) } 1 - \cos 6x = 2 \sin^2 3x$$

$$\text{g) } \frac{1 - \cos x}{\sin x} = \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \tan \frac{x}{2}$$

$$\text{r) } \frac{1 + \cos 2x}{2 \cos x} = \frac{2 \cos^2 x}{2 \cos x} = \cos x$$

2/4/2012 12:4

$$\text{uf) } \frac{1 + \cos 2x}{1 - \cos 2x} = \frac{2 \cos^2 x}{2 \sin^2 x} = \cot^2 x$$

$$\text{p) } \frac{\sin 2x}{1 + \cos 2x} = \frac{2 \sin x \cos x}{2 \cos^2 x} = \frac{\sin x}{\cos x} = \tan x$$

$$\text{g) } \frac{1 + \cos 4x}{\cos^2 x - \sin^2 x} = \frac{2 \cos^2 2x}{\cos 2x} = 2 \cos 2x$$

$$\text{r) } \frac{1 - \cos 3x}{1 + \cos 3x} = \frac{2 \sin^2 \frac{3x}{2}}{2 \cos^2 \frac{3x}{2}} = \tan^2 \frac{3x}{2}$$

2/12/2015

$$u) 1 + \sin \alpha = 2 \cos^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

$$1 + \sin \alpha = 1 + \cos \left(\frac{\pi}{2} - \alpha \right) = 2 \cos^2 \frac{1}{2} \left(\frac{\pi}{2} - \alpha \right) =$$

$$= 2 \cos^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

$$f) 1 - \sin \alpha = 2 \sin^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

$$1 - \sin \alpha = 1 - \cos \left(\frac{\pi}{2} - \alpha \right) = 2 \sin^2 \frac{1}{2} \left(\frac{\pi}{2} - \alpha \right) =$$

$$= 2 \sin^2 \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$$

$$g) \frac{\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}} = \frac{1}{\cos \alpha} - \operatorname{tg} \alpha$$

$$\frac{\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}} = \frac{\frac{\cos^2 \frac{\alpha}{2}}{2} - \frac{\sin^2 \frac{\alpha}{2}}{2}}{\frac{\cos^2 \frac{\alpha}{2}}{2} + \frac{\sin^2 \frac{\alpha}{2}}{2}} + \frac{2 \cos \frac{\alpha}{2} \sin \frac{\alpha}{2}}{2}$$

$$= \frac{\cos \alpha}{1 + \sin \alpha} = \frac{\cos \alpha}{1 + \sin \alpha}$$

$$= \frac{\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} - 2 \cos \frac{\alpha}{2} \sin \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}} = \frac{1 - \sin \alpha}{\cos \alpha} = \frac{1}{\cos \alpha} - \operatorname{tg} \alpha$$

$$2) \cot \frac{\Delta}{2} = \frac{1 + \cos \Delta}{\sin \Delta} = 2 \cot \frac{\Delta}{2}$$

$$\frac{\sin \Delta}{1 - \cos \Delta} = \frac{1 - \cos \Delta}{\sin \Delta}$$

$$= \frac{\sin^2 \Delta - (1 - \cos \Delta)^2}{\sin \Delta (1 - \cos \Delta)} = \frac{\sin^2 \Delta - 1 + \cos^2 \Delta + 2 \cos \Delta}{\sin \Delta (1 - \cos \Delta)}$$

$$= \frac{2 \cos \Delta - 1 + \cos 2\Delta}{\sin \Delta (1 - \cos \Delta)} = \frac{2 \cos \Delta \sin^2 \frac{\Delta}{2}}{\sin \Delta (1 - \cos \Delta)}$$

$$= \frac{2 \cos \Delta + 2 \sin^2 \frac{\Delta}{2} - \cos \Delta}{1 + \cos \Delta}$$

$$1 + \cos \Delta$$

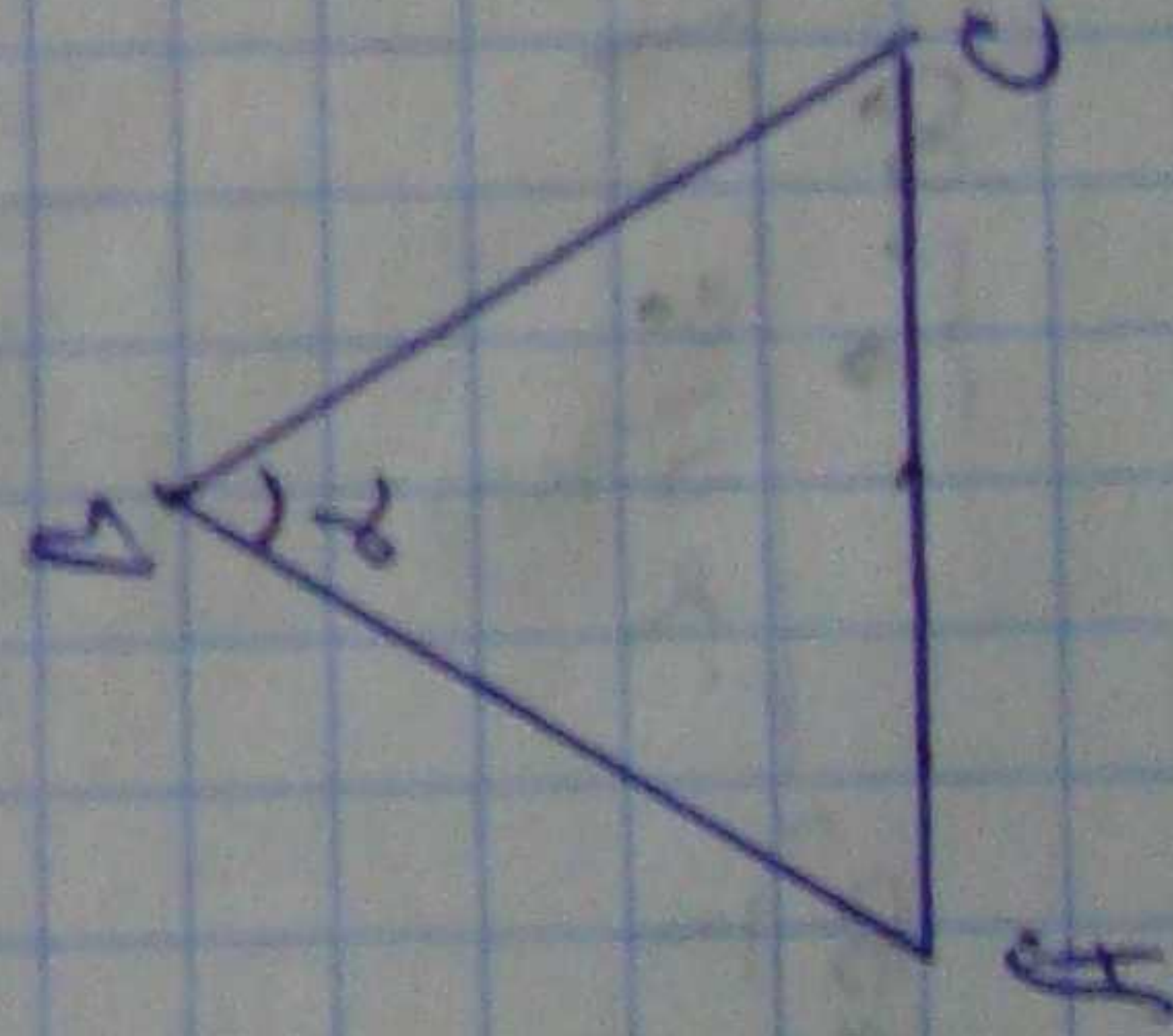
$$\cot \frac{\Delta}{2} = \frac{1 + \cos \Delta}{\sin \Delta} = \frac{1 - \cos \Delta}{\sin \Delta}$$

$$= \frac{1 + \cos \Delta - 1 + \cos \Delta}{\sin \Delta} = \frac{2 \cos \Delta}{\sin \Delta} = 2 \cot \frac{\Delta}{2}$$

Pulchra 126

$$\cos \Delta = \frac{1}{9}$$

$$\text{quadrant } \cos \left(\frac{180 - \Delta}{2} \right) = -a$$



$$2) \operatorname{ctg} \frac{\alpha}{2} = \operatorname{tg} \frac{\alpha}{2} = 2 \operatorname{ctg} \alpha$$

$$\frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

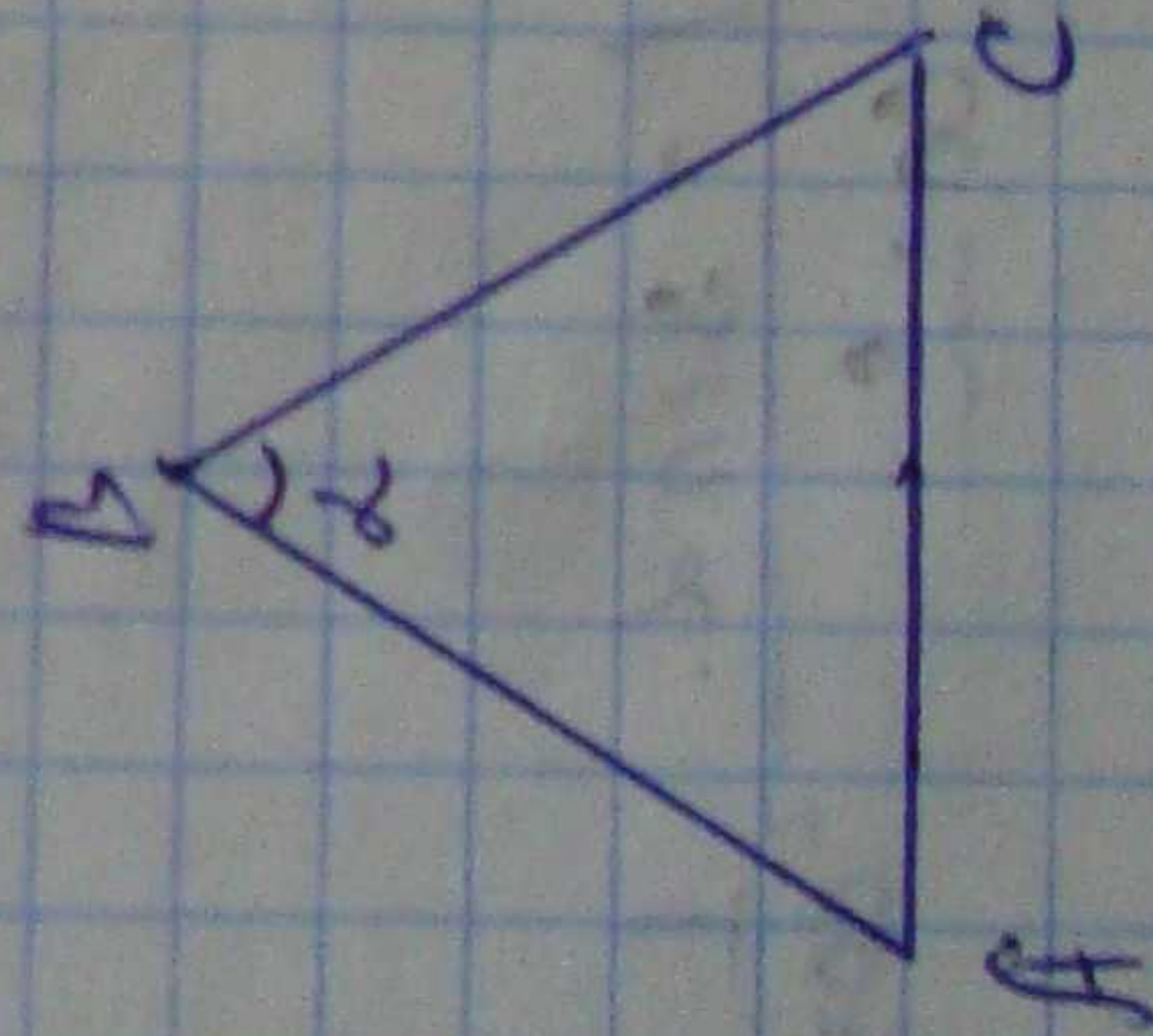
$$= \frac{\sin^2 \alpha - (1 - \cos \alpha)^2}{\sin \alpha (1 - \cos \alpha)} = \frac{\sin^2 \alpha - 1 + \cos^2 \alpha + 2 \cos \alpha}{\sin \alpha (1 - \cos \alpha)}$$

$$= \frac{2 \cos \alpha - 1 + \cos 2\alpha}{\sin \alpha (1 - \cos \alpha)} = \frac{2 \cos \alpha \sin^2 \frac{\alpha}{2}}{\sin \alpha (1 - \cos \alpha)}$$

$$= \frac{2 \cos \alpha + 2 \sin^2 \frac{\alpha}{2} - \cos \alpha}{1 - \cos \alpha}$$

$$\operatorname{ctg} \frac{\alpha}{2} = \operatorname{tg} \frac{\alpha}{2} = \frac{1 + \cos \alpha}{\sin \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$= \frac{1 + \cos \alpha - 1 + \cos \alpha}{\sin \alpha} = \frac{2 \cos \alpha}{\sin \alpha} = 2 \operatorname{ctg} \alpha$$



Решение 126

$$\cos \alpha = \frac{1}{9}$$

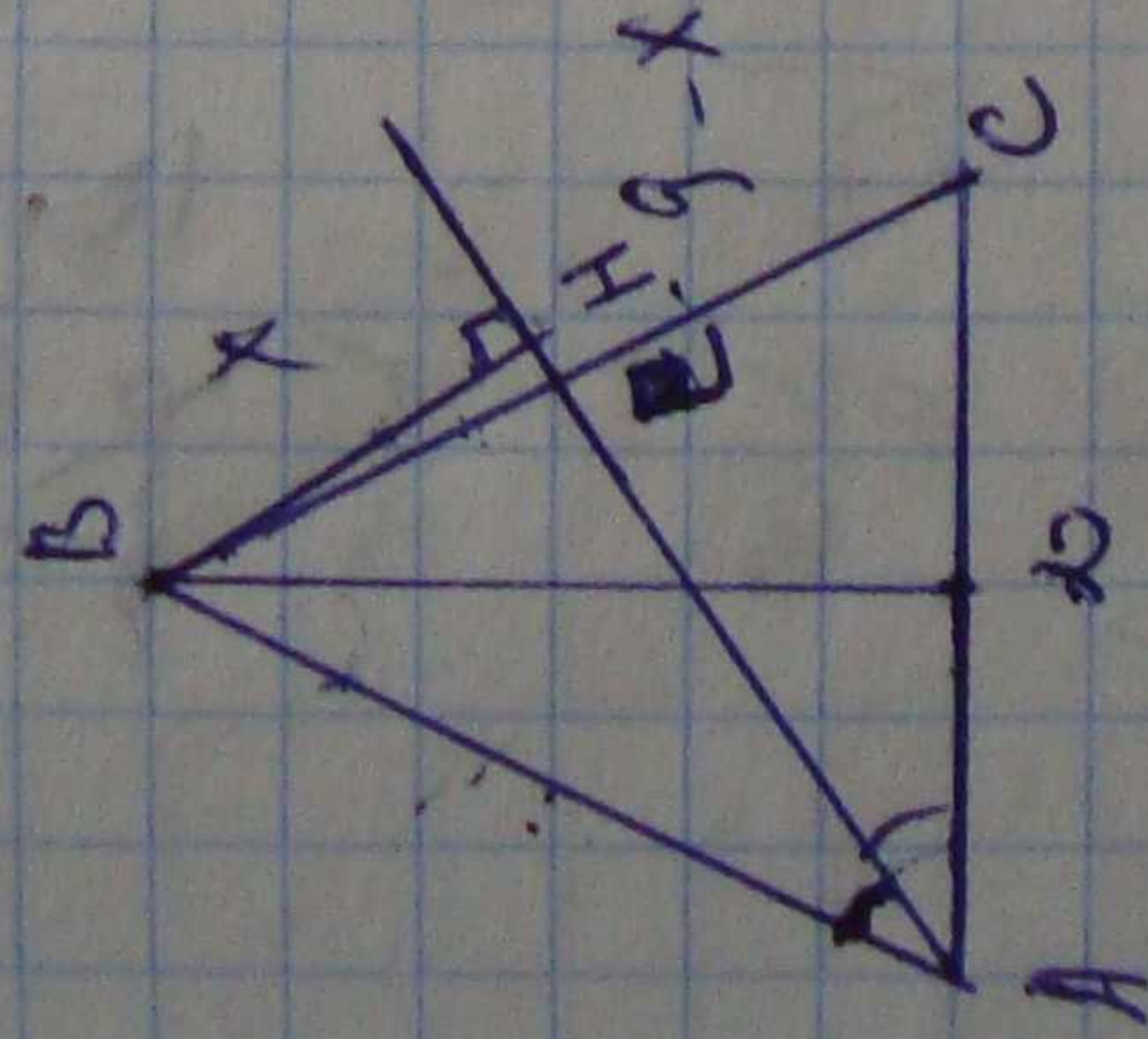
$$\operatorname{tg} \frac{\alpha}{2} = \cos \left(\frac{180^\circ - \alpha}{2} \right) = \frac{1}{9}$$

$$\cos\left(90 - \frac{\angle}{2}\right) = \sin \frac{\angle}{2}$$

$$\left(\sin \angle = \sqrt{1 - \frac{1}{81}}\right) = \sqrt{\frac{80}{81}} = \frac{4\sqrt{5}}{9}$$

$$\sin \frac{\angle}{2} = \sqrt{\frac{1 - \cos \angle}{2}} = \frac{2}{3}$$

$$\Rightarrow \cos\left(90 - \frac{\angle}{2}\right) = \sin \frac{\angle}{2} = \frac{2}{3}$$



For 127.

$$AB = BE = 9$$

$$AC = 14$$

$$\sin \frac{\angle}{2} = \cos\left(90 - \frac{\angle}{2}\right)$$

$$\cos\left(\frac{180^\circ - \angle}{2}\right) = \cos(45^\circ - \angle) = \frac{\sqrt{2}}{2} (\cos \angle + \sin \angle)$$

$$\sin \angle = \frac{3}{9}$$

$$\cos \angle = \frac{7}{9}$$

$$\cos \frac{\angle}{2} =$$

$$= \sqrt{\frac{1 + \cos \angle}{2}} = \sqrt{\frac{1 + \frac{7}{9}}{2}} = \sqrt{\frac{\frac{16}{9}}{2}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\frac{AB}{AC} = \frac{BE}{EC} \Rightarrow x = \frac{(9-x) \cdot 9}{14} = \frac{81 - 9x}{14}$$

$$14x + 9x = 81$$

$$23x = 81$$

$$x = \frac{81}{23}$$

$$BH = \sin \frac{\alpha}{2} \cdot AB = \frac{2\sqrt{2}}{3} \cdot 9 = 6\sqrt{2}$$

$$= \frac{1}{3} \cdot 9 = 3$$

$\pi \cdot 3$:

$\frac{2}{\alpha} \cdot 128$

$$\text{we } \sin \alpha = \frac{3}{5} \quad \text{to } \frac{5\pi}{2} < \alpha < \frac{3\pi}{2} \quad \text{or } \frac{\pi}{2} < \alpha < \pi$$

II for

$$\text{Quantity } \sqrt{10} \sin \frac{\alpha}{2}$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\sin \alpha = \frac{3}{5} \Rightarrow \cos \alpha = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{10}$$

$$\sqrt{10} \sin \frac{\alpha}{2} = \sqrt{10} \cdot \frac{3}{\sqrt{10}} = 3$$

$$P) \sin \alpha = \frac{\sqrt{15}}{8} \quad \text{to } \pi < \alpha < \frac{3}{2}\pi$$

$$4 \cos \frac{\alpha}{2}$$

III for.

$$4 \cos \frac{\alpha}{2} - 4 \sqrt{\frac{1 + \frac{8}{\alpha}}{2}} - \sqrt{15}$$

$$4 \cos \frac{\pi}{2} - ?$$

$$\cos \alpha = \frac{1}{\sqrt{1 + \frac{49}{64}}} = \frac{8}{\sqrt{115}}$$

$$\cos \frac{\pi}{2} = \sqrt{\frac{1 + \sqrt{1 + \frac{1}{2}}}{2}} = \frac{1}{2}$$

$$4 \cos \frac{\pi}{2} = -1$$

Temp: -1:

9) $\sin \alpha = -\frac{\sqrt{15}}{8}$ $\frac{3}{2} \pi < \alpha < 2\pi$ $\Rightarrow \alpha \in \underline{\overline{15}}^{\text{Quadr.}}$

Qantali $3\sqrt{15} - 29\frac{2}{2}$

$$\cos \alpha = \sqrt{1 - \frac{15}{64}} = \frac{7}{8}$$

$$3\sqrt{15} + \lg \frac{2}{2} = 3\sqrt{15} = \frac{1 - \frac{7}{8}}{\frac{\sqrt{15}}{8}} = \frac{1}{\sqrt{15}} \cdot \left(-\frac{1}{\sqrt{15}}\right) = -3$$

Then: $3\sqrt{15} + \lg \frac{2}{2} = -3$

2/12/2019

$$\text{we } \tan \frac{\pi}{12} + \cot \frac{\pi}{12} = 4$$

$$\frac{1 - \cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} + \frac{1 + \cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} = \frac{1 - \cos \frac{\pi}{6} + 1 + \cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} = 2$$

$$= \frac{2}{\sin \frac{\pi}{6}} = 4$$

$$F) \cot \frac{\pi}{8} - \tan \frac{\pi}{8} = 2$$

$$\cot \frac{\pi}{8} - \tan \frac{\pi}{8} = \frac{1 + \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} - \frac{1 - \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} = 2$$

$$= \frac{2 \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} = 2 \cot \frac{\pi}{4} = 2$$

$$g) \frac{\cot \frac{\pi}{12} - \tan \frac{\pi}{12}}{\cot \frac{\pi}{8} + \tan \frac{\pi}{8}} = \sqrt{3 - 2}$$

$$\frac{1 + \cos \frac{\pi}{6} - 1 + \cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} \cdot \frac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4} + 1 - \cos \frac{\pi}{4}} = 2$$

$$= 2 \cot \frac{\pi}{6} \cdot \frac{\sin \frac{\pi}{4}}{2} = \cot \frac{\pi}{6} \cdot \sin \frac{\pi}{4} = \sqrt{3} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2}$$

$$7) \frac{\operatorname{tg} \frac{\pi}{12} - \operatorname{ctg} \frac{\pi}{12}}{\operatorname{tg} \frac{\pi}{8} - \operatorname{ctg} \frac{\pi}{8}} = \sqrt{3}$$

$$\frac{1 - \cos \frac{\pi}{6} - 1 - \cos \frac{\pi}{6}}{1 - \cos \frac{\pi}{4} - 1 - \cos \frac{\pi}{4}} \cdot \frac{2 \cos \frac{\pi}{6}}{2 \cos \frac{\pi}{4}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{\sqrt{2}}$$

$$\frac{1 - \cos \frac{\pi}{6} - 1 - \cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} \cdot \frac{\sin \frac{\pi}{4}}{1 - \cos \frac{\pi}{4} - 1 - \cos \frac{\pi}{4}}$$

$$\frac{\cos \frac{\pi}{6}}{\cos \frac{\pi}{4}} \cdot \frac{\sin \frac{\pi}{4}}{\sin \frac{\pi}{6}} = \frac{\cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} = \operatorname{ctg} \frac{\pi}{6} = \sqrt{3}$$

Answer: $\sqrt{3}$

Answer: 130

$$\operatorname{ctg} \alpha = -\frac{\sqrt{2}}{4} \quad \text{if} \quad \frac{5}{2}\pi < \alpha < 3\pi \quad \text{find} \quad \frac{\pi}{2} < \alpha < \pi \Rightarrow \alpha \in \left(\frac{\pi}{2}, \pi\right) \text{ given}$$

$$\sin 2\alpha = \sqrt{6} \sin \frac{\alpha}{2} = e$$

$$\operatorname{ctg} \alpha = -\frac{\sqrt{2}}{4} \Rightarrow \operatorname{tg} \alpha = -\frac{4}{\sqrt{2}} = -2\sqrt{2}$$

$$\sin \alpha = \cos \beta \Rightarrow \sqrt{\frac{1}{1+\operatorname{tg}^2 \alpha}} = \sqrt{\frac{1}{1+8}} = \frac{1}{3}$$

$$\sqrt{6} \sin \frac{\alpha}{2} = \sqrt{6} \cdot \sqrt{\frac{1 - \cos \alpha}{2}} = \sqrt{6} \cdot \sqrt{\frac{2}{3}} = \sqrt{2} = e$$

$$f) \operatorname{tg} \lambda = \frac{6\sqrt{2}}{5} \quad \text{u} \quad 0 < \lambda < \frac{\pi}{2} \quad \text{u} \quad \lambda \in I \text{ given}$$

$$\sqrt{11} \cos \frac{\lambda}{2} = c \quad ?$$

$$\cos \lambda = \sqrt{\frac{1}{1 + \operatorname{tg}^2 \lambda}} = \sqrt{\frac{49}{72}}$$

$$= \sqrt{1 + \frac{49}{49}} = \frac{7}{11}$$

$$\sqrt{11} \cos \frac{\lambda}{2} = \sqrt{11} \cdot \sqrt{\frac{1 + \cos \lambda}{2}} = \sqrt{11} \cdot \sqrt{\frac{19}{22}} = \sqrt{\frac{19}{2}}$$

$$g) \sqrt{118} \sin \frac{\lambda}{2} = c, \quad \operatorname{tg} \lambda = -\frac{24\sqrt{6}}{5} \quad \text{u} \quad \frac{5}{2}\pi < \lambda < 3\pi$$

$$\cos \lambda = \sqrt{\frac{1}{1 + \operatorname{tg}^2 \lambda}}$$

$$= \sqrt{\frac{1}{1 + \frac{3456}{25}}} = \sqrt{\frac{25}{3481}} = \frac{5}{59}$$

$$\sqrt{118} \sin \frac{\lambda}{2} = \sqrt{118} \sqrt{\frac{1 + \frac{5}{59}}{2}} = \sqrt{\frac{32}{59}} = \frac{\sqrt{2}}{\sqrt{118}}$$

$$= 8$$

7.4.8

day 131

$$|\cos \lambda + \frac{1}{5}| = \frac{3}{5}$$

$$k \quad \frac{5\pi}{2} < \lambda < 3\pi \quad \pi < \lambda < \frac{9\pi}{2}$$

$$\text{given } \sqrt{10} \cos \frac{\lambda}{2} = -\frac{1}{5}$$

$$\frac{\pi}{2} < \frac{\lambda}{2} < \frac{3\pi}{2}$$

$$\cos \lambda + \frac{1}{5} = \frac{3}{5} \quad 0$$

$$\cos \lambda = \frac{2}{5} \quad -\frac{1}{5}$$

$$\left\{ \cos \lambda = -\frac{2}{5} \right.$$

$$\left\{ \cos \lambda + \frac{1}{5} = \frac{3}{5} \right.$$

$$\left\{ \cos \lambda = \frac{2}{5} \right.$$

$$\frac{1}{5} - \cos \lambda = \frac{3}{5}$$

$$\cos \lambda \in \left[-\frac{1}{5}, \frac{1}{5} \right]$$

$$\cos \lambda = \frac{2}{5}$$

$$\cos \lambda \in \left(-\frac{1}{5}, \frac{1}{5} \right)$$

$$\cos \lambda = -\frac{2}{5}$$

first

$$\pi < \lambda < \frac{3\pi}{2}, \text{ then } \cos \lambda = -\frac{2}{5}$$

$$\sqrt{10} \cos \frac{\lambda}{2} = \sqrt{10} \cdot \sqrt{\frac{1 + \cos \lambda}{2}} = \sqrt{10} \cdot \sqrt{\frac{1 - \frac{2}{5}}{2}} = \sqrt{10} \cdot \sqrt{\frac{3}{10}} = \sqrt{3}$$

$$\frac{\pi}{2} < \frac{\lambda}{2} < \frac{3\pi}{2}$$

$$P) \left| \cos \lambda - \frac{3}{25} \right| \geq \frac{1}{5} \Leftrightarrow \frac{5}{2} \pi < \lambda < 3\pi \quad \text{for } \frac{\pi}{2} < \lambda < \pi$$

$$\text{Range of } \sin \frac{\lambda}{2} \text{ is } \left[\frac{1}{2}, 1 \right]$$

$$\cos \lambda - \frac{3}{25} \geq 0$$

$$\cos \lambda \geq \frac{3}{25}$$

$$\left\{ \begin{array}{l} \cos \lambda \geq \frac{3}{25} \\ \cos \lambda - \frac{3}{25} \geq \frac{1}{5} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \cos \lambda \in \left[\frac{3}{25}, 1 \right] \\ \cos \lambda \geq \frac{8}{25} \end{array} \right. \Rightarrow \cos \lambda \in \left[\frac{8}{25}, 1 \right]$$

$$\left\{ \begin{array}{l} \cos \lambda < \frac{3}{25} \\ \frac{3}{25} - \cos \lambda = \frac{1}{5} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \cos \lambda \in \left(-\infty, \frac{3}{25} \right) \\ \cos \lambda = \frac{2}{25} \end{array} \right.$$

$$\cos \lambda = \frac{2}{25}$$

$$\sin \frac{\lambda}{2} = \sqrt{\frac{1 - \cos \lambda}{2}} = \sqrt{\frac{1 - \frac{2}{25}}{2}} = \sqrt{\frac{23}{50}}$$

Range

$$Q) \sqrt{2} \tan \frac{\lambda}{2} = 0, \text{ bpt } \left| \cos \lambda - \frac{1}{6} \right| \geq \frac{1}{2} \Leftrightarrow \pi < \lambda < \frac{3}{2}\pi$$

$$\left| \cos \lambda - \frac{1}{6} \right| \geq \frac{1}{2}$$

$$\cos \lambda - \frac{1}{6} \geq 0 \quad \cos \lambda \geq \frac{1}{6}$$

$$P) \left| \cos \lambda - \frac{3}{25} \right| \leq \frac{1}{5} \Leftrightarrow \frac{5}{4} \pi < \lambda < 3\pi \quad \text{for } \frac{\pi}{2} < \lambda < \pi$$

$$\text{Range of } \sin \frac{\lambda}{2} = [-1, 1]$$

$$\cos \lambda - \frac{3}{25} \geq 0$$

$$\cos \lambda \geq \frac{3}{25}$$

$$\left\{ \begin{array}{l} \cos \lambda \geq \frac{3}{25} \\ \cos \lambda - \frac{3}{25} \geq \frac{1}{5} \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \cos \lambda \in \left[\frac{3}{25}, +\infty \right) \\ \cos \lambda \geq \frac{8}{25} \end{array} \right. \Rightarrow \cos \lambda \geq \frac{8}{25}$$

$$\left\{ \begin{array}{l} \cos \lambda < \frac{3}{25} \\ \frac{3}{25} - \cos \lambda = \frac{1}{5} \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \cos \lambda \in (-\infty, \frac{3}{25}) \\ \cos \lambda = \frac{2}{25} \end{array} \right. \Rightarrow \cos \lambda = \frac{2}{25}$$

$$\cos \lambda = \frac{2}{25}$$

$$5\sqrt{6} \sin \frac{\lambda}{2} = 5\sqrt{6} \cdot \sqrt{\frac{1 - \cos \lambda}{2}}$$

$$= \frac{\sqrt{3}}{5\sqrt{6}} \sqrt{\frac{24}{2 \cdot 25}} = \frac{2}{5}$$

for 9.

$$9) \sqrt{2} \tan \frac{\lambda}{2} = 0, \text{ bpt } \left| \cos \lambda - \frac{1}{6} \right| \leq \frac{1}{2} \Leftrightarrow \pi < \lambda < \frac{3}{2}\pi$$

$$\left| \cos \lambda - \frac{1}{6} \right| \leq \frac{1}{2}$$

$$\cos \lambda - \frac{1}{6} \geq 0 \quad \cos \lambda \geq \frac{1}{6}$$

$$\left\{ \cos \lambda \geq \frac{1}{6} \right. \\ \left. \cos \lambda - \frac{1}{6} \geq \frac{1}{2} \right\}$$

$$\Rightarrow \left\{ \cos \lambda \in \left[\frac{1}{6}, +\infty \right) \right. \\ \left. \cos \lambda \geq \frac{2}{3} \right\} \Rightarrow \cos \lambda \geq \frac{2}{3}$$

$$\cos \lambda < \frac{1}{6}$$

$$\cos \lambda \in \left(-\infty, \frac{1}{6} \right)$$

$$\left\{ \frac{1}{6} - \cos \lambda \geq \frac{1}{2} \right\}$$

$$\cos \lambda \geq \frac{1}{3}$$

$$\Rightarrow \cos \lambda \geq \frac{1}{3}$$

$$\cos \lambda \geq \frac{1}{3}$$

$$- \sqrt{2} \tan \frac{\lambda}{2} \geq \sqrt{2} \sqrt{\frac{1 - \cos \lambda}{1 + \cos \lambda}} \cdot \sqrt{2} \sqrt{\frac{4}{3} \cdot \frac{3}{2}} \quad \sqrt{2} \cdot \sqrt{2} = 2$$

$$\Rightarrow m: -2$$

quy định 132

$$\Rightarrow \tan \frac{\lambda}{2} \geq 3\sqrt{7} \tan \frac{\lambda}{2}, \text{ bpt } \cos 2\lambda \geq \frac{1}{8} \quad 4\pi < \lambda < \frac{3}{2}\pi$$

$$\frac{\pi}{2} < \frac{\lambda}{2} < \frac{3}{4}\pi$$

$$\cos \lambda \geq \pm \sqrt{\frac{1 + \cos}{2}}$$

$$\cos \lambda \geq - \sqrt{\frac{1 + \cos 2\lambda}{2}} \geq - \sqrt{\frac{9}{16}} \geq - \frac{3}{4}$$

$$3\sqrt{7} \tan \frac{\lambda}{2} \geq -3\sqrt{7} \sqrt{\frac{1 + \frac{3}{4}}{1 - \frac{3}{4}}} \geq -3\sqrt{7} \cdot \sqrt{7} \geq -21$$

$$\Rightarrow m: -21$$

$$4P) \text{ nur } \frac{1}{2} \cdot 4\sqrt{15} \cdot \lg \frac{1}{2} \text{ re, } \lg \cos 2\alpha = \frac{17}{32} \quad 4$$

$$\pi < \alpha < \frac{3}{2}\pi$$

$$\frac{\pi}{2} < \frac{\alpha}{2} < \frac{3}{4}\pi$$

$$\cos \alpha = 2 \cos^2 \frac{\alpha}{2} - 1$$

$$1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\cos \alpha = \sqrt{\frac{1 + \cos 2\alpha}{2}} = \sqrt{\frac{49}{64}} = \frac{7}{8}$$

$$4\sqrt{15} \cdot \lg \frac{1}{2} = -4\sqrt{15} \cdot \sqrt{\frac{1 + \frac{7}{8}}{1 - \frac{7}{8}}} = -\sqrt{15} \cdot 4\sqrt{15} =$$

$$= -60$$

~~Step 133~~

$$0,25x + 1 = 0,4x$$

$$0,15x = 1$$

$$x = \frac{1}{0,15}$$

2/упр 137.

$$\begin{aligned} \text{в) } \sin 37,5^\circ \cdot \sin 7,5^\circ &= \frac{1}{2} (\cos(37,5^\circ + 7,5^\circ) - \cos(37,5^\circ - 7,5^\circ)) = \\ &= \frac{1}{2} (\cos 45^\circ - \cos 30^\circ) = \frac{1}{2} \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \right) = \frac{\sqrt{2} - \sqrt{3}}{4} \end{aligned}$$

$$\begin{aligned} \text{г) } \sin 37,5^\circ \cdot \sin 7,5^\circ &= \frac{1}{2} (\cos(37,5^\circ - 7,5^\circ) - \cos(37,5^\circ + 7,5^\circ)) = \\ &= \frac{1}{2} \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \right) = \frac{\sqrt{3} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{д) } \cos 37,5^\circ \cos 7,5^\circ &= \frac{1}{2} (\cos(37,5^\circ + 7,5^\circ) + \cos(37,5^\circ - 7,5^\circ)) = \\ &= \frac{1}{2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \right) = \frac{\sqrt{2} + \sqrt{3}}{4} \end{aligned}$$

$$\begin{aligned} \text{е) } \sin 37,5^\circ \cos 7,5^\circ &= (\sin 37,5^\circ \cdot \sin 82,5^\circ) = \frac{1}{2} (\cos \\ &= \frac{1}{2} (\sin 45^\circ + \sin 30^\circ) = \frac{1}{2} \cdot \left(\frac{\sqrt{2}}{2} + \frac{1}{2} \right) = \frac{\sqrt{2} + 1}{4} \end{aligned}$$

$$\text{ж) } \sin 75^\circ \sin 15^\circ = \frac{1}{2} (\cos 60^\circ - \cos 90^\circ) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

2/упр 138

$$\text{в) } \sin 75^\circ \cos 15^\circ = \frac{1}{2} (\sin 90^\circ + \sin 60^\circ) = \frac{2 + \sqrt{3}}{4}$$

$$\begin{aligned} \text{д) } \cos 67,5^\circ \sin 22,5^\circ &= \frac{1}{2} (\sin 90^\circ - \sin 45^\circ) = \frac{1}{2} \cdot \left(1 - \frac{\sqrt{2}}{2} \right) = \\ &= \frac{2 - \sqrt{2}}{4} \end{aligned}$$

$$9) \cos 67,5^\circ \cos 22,5^\circ = \frac{1}{2} (\cos 90^\circ + \cos 45^\circ) = \frac{1}{2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right)$$

$$11) \cos 97,5^\circ \cos 34,5^\circ = \frac{1}{2} (\cos 135^\circ + \cos 60^\circ) =$$

$$= \frac{1}{2} \left(\frac{1}{2} - \frac{\sqrt{2}}{2} \right) = \frac{1 - \sqrt{2}}{4}$$

2/uptd 139

$$12) \cos \frac{5\pi}{12} \cos \frac{\pi}{4} = \frac{1}{2} \left(\cos \left(\frac{5\pi}{12} + \frac{\pi}{4} \right) + \cos \left(\frac{5\pi}{12} - \frac{\pi}{4} \right) \right) =$$

$$= \frac{1}{2} \left(\cos \frac{2\pi}{3} + \cos \frac{\pi}{6} \right) = \frac{1}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) = \frac{\sqrt{3} - 1}{4}$$

$$13) \cos \frac{5\pi}{12} \sin \frac{\pi}{4} = \frac{1}{2} \left(\sin \left(\frac{5\pi}{12} + \frac{\pi}{4} \right) - \sin \left(\frac{5\pi}{12} - \frac{\pi}{4} \right) \right) =$$

$$= \frac{1}{2} \left(\sin \frac{2\pi}{3} - \sin \frac{\pi}{6} \right) = \frac{1}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) = \frac{\sqrt{3} - 1}{4}$$

$$14) \sin \frac{\pi}{12} \cos \frac{\pi}{4} = \frac{1}{2} \left(\sin \left(\frac{\pi}{12} + \frac{\pi}{4} \right) + \sin \left(\frac{\pi}{12} - \frac{\pi}{4} \right) \right) =$$

$$= \frac{1}{2} \left(\sin \frac{2\pi}{3} + \sin \left(-\frac{\pi}{12} \right) \right) = \frac{1}{2} \left(\sin \frac{2\pi}{3} - \sin \frac{\pi}{12} \right) =$$

$$= \frac{1}{2} \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2} - \sqrt{3}}{2} \right) = \frac{\sqrt{3}(2 - \sqrt{3})}{4}$$

$$\begin{aligned}
 6) \quad \sin \frac{\pi}{12} \cdot \cos \frac{\pi}{4} &= \frac{1}{2} \left(\sin \frac{\pi}{8} + \sin \left(\frac{\pi}{8} \right) \right) = \\
 &= \frac{1}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) = \frac{\sqrt{3}+1}{4} = \frac{\sqrt{3}-1}{4}
 \end{aligned}$$

$$7) \quad \sin \frac{\pi}{12} \cdot \sin \frac{\pi}{4} = \frac{1}{2} \left(\cos \frac{\pi}{6} - \cos \frac{\pi}{3} \right) = \frac{1}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) =$$

$$= \frac{\sqrt{3}-1}{4}$$

2/4/2014

$$\text{w) } \sin x + \sin 5x = 2 \left(\cos(-4x) - \cos 6x \right) =$$

$$\begin{aligned}
 &= 2 \left(1 + \cos 4x + \cos 6x \right) = 2 \left(2 \cos^2 2x - 1 + 2 \cos^2 3x - 1 \right) \\
 &= (-4) \left(\cos^2 2x + \cos^2 3x - 1 \right) \Rightarrow = -2 \sin 2x \cos 2x
 \end{aligned}$$

$$\text{p) } \sin 8x - \sin 2x = 2 \sin 3x \cos 5x$$

$$9) \quad \cos 3y - \cos y = -2 \sin 2y \sin y$$

$$7) \quad \cos 2x + \cos 4x = 2 \cos 3x \cos(-x) = 2 \cos 3x \cos x$$

2/4/2014

$$\text{w) } \sin x + \cos 3x = \cos \left(\frac{\pi}{2} - x \right) + \cos 3x =$$

$$= 2 \cos \left(\frac{\pi + 2x}{2} \right) \cos \left(\frac{\pi - 4x}{2} \right) =$$

$$= 2 \cos \left(\frac{\pi}{4} + x \right) \cos \left(\frac{\pi}{4} - 2x \right) = 2 \cos \left(\frac{\pi}{4} - 2x \right) \cos \left(\frac{\pi}{4} + x \right) = \frac{\sqrt{2}}{2} (\cos 2x + \sin 2x) =$$

$$= (\cos x - \sin x) (\cos 2x + \sin 2x)$$

$$P) \sin 2x - \cos 2y = \cos(\pi$$

$$= 2 \sin x \cos y - 2 \cos^2 y + 1 =$$

$$= 2 \cos y (\sin x - \cos y) + 1$$

$$P) \sin 2x - \cos 2y = 2 \sin x \cos x - \cos 2y =$$

$$= 2 \cos \left(\frac{\pi}{2} - x \right) \cos x - \cos 2y =$$

$$= 2 \left(\cos \frac{\pi}{2} + \cos \left(\frac{\pi}{2} - 2x \right) \right) - \cos 2y =$$

$$= \cos \left(\frac{\pi}{2} - 2x \right) - \cos 2y =$$

$$= -2 \sin \left(\frac{\pi}{2} - 2x - 2y \right) \sin \left(\frac{\pi}{2} - 2x + 2y \right) =$$

$$= -2 \sin \left(\frac{\pi - 4(x+y)}{4} \right) \sin \left(\frac{\pi + 4(x-y)}{4} \right) =$$

$$= -2 \sin \left(\frac{\pi}{4} - x - y \right) \sin \left(\frac{\pi}{4} - x + y \right) =$$

$$= 2 \sin \left(x + y - \frac{\pi}{4} \right) \sin \left(y - x + \frac{\pi}{4} \right)$$

$$7) \cos y + \sin 3y = \cos y + \cos\left(\frac{\pi}{2} - 3y\right) =$$

$$= 2 \cos\left(\frac{4y - \frac{\pi}{2}}{2}\right) \cos\left(\frac{\frac{\pi}{2} - 2y}{2}\right) =$$

$$= 2 \cos\left(2y - \frac{\pi}{4}\right) \cos\left(\frac{\pi}{4} - y\right)$$

$$8) \cos 4x - \sin 2y = \cos 4x - \cos\left(\frac{\pi}{2} - 2y\right) =$$

$$= -2 \sin\left(\frac{4x - \frac{\pi}{2} + 2y}{2}\right) \sin\left(\frac{4x + \frac{\pi}{2} - 2y}{2}\right) =$$

$$= -2 \sin\left(2x + y - \frac{\pi}{4}\right) \sin\left(2x - y + \frac{\pi}{4}\right) =$$

$$= 2 \sin\left(\frac{\pi}{4} - 2x - y\right) \sin\left(\frac{\pi}{4} + 2x - y\right)$$

Answer 142

$$\text{we } \sin 12^\circ + \sin 24^\circ = 2 \sin 18^\circ \cos 6^\circ$$

$$9) \cos \frac{11\pi}{12} + \cos \frac{3\pi}{4} = 2 \cos\left(\frac{5\pi}{6}\right) \cos \frac{1\pi}{12} = 2 \cdot \frac{1}{2} \cdot \cos \frac{\pi}{12}$$

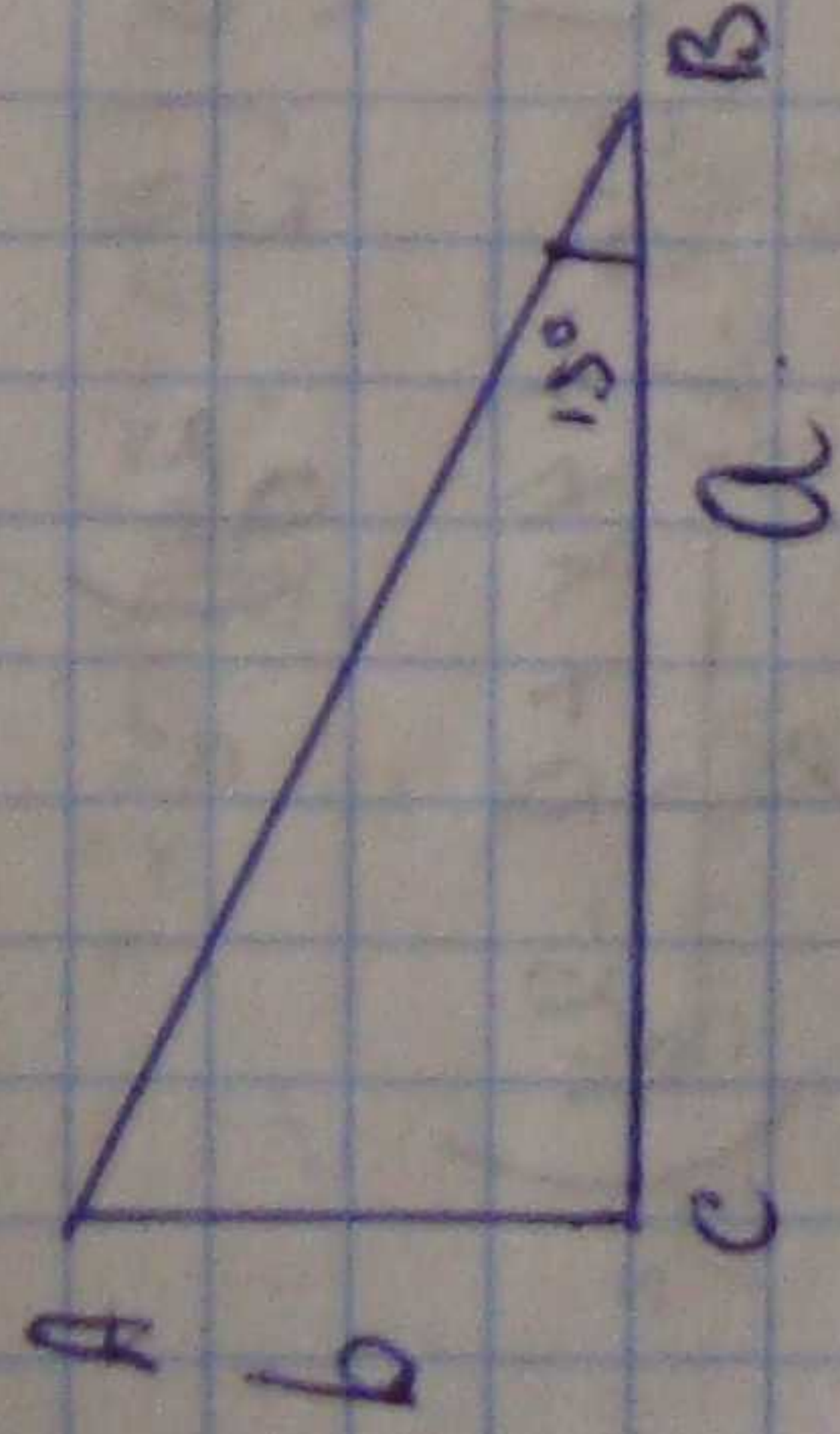
$$9) \cos \frac{\pi}{6} - \cos \frac{7\pi}{12} = -2 \sin \frac{3\pi}{8} \sin \frac{5\pi}{24} = -2 \frac{\sqrt{2}}{2} \cdot \cos \frac{5\pi}{12} =$$

$$= -\sqrt{2} \cos \frac{5\pi}{12}$$

$$n) \sin \frac{2\pi}{5} + \sin \frac{\pi}{5} = 2 \sin \left(\frac{2\pi}{5} + \frac{\pi}{5} \right) \cos \left(\frac{2\pi}{5} - \frac{\pi}{5} \right)$$

$$= 2 \sin \frac{3\pi}{10} \cos \frac{\pi}{10}$$

Examp 143



$$\angle = 15^\circ$$

$$\begin{cases} a+b = \sqrt{54} \\ a-b = \sqrt{50} \end{cases}$$

$$\begin{cases} a = \frac{\sqrt{54} - b}{2} \\ \sqrt{54} - b - b = \sqrt{50} \end{cases} \Rightarrow \begin{cases} a = \frac{\sqrt{54}(\sqrt{54}-1) + \sqrt{50}}{2} \\ b = \frac{\sqrt{54} - \sqrt{50}}{2} \end{cases}$$

$$c^2 = a^2 + b^2 = (\sqrt{54}(\sqrt{54}-1) + \sqrt{50})^2 + (\sqrt{54} - \sqrt{50})^2 =$$

$$= 54(54 - 2\sqrt{54} + 1) + 50 + 2\sqrt{54} =$$

$$c^2 = a^2 + b^2 = \left(\frac{\sqrt{54} + \sqrt{50}}{2} \right)^2 + \left(\frac{\sqrt{54} - \sqrt{50}}{2} \right)^2 =$$

$$= \frac{(\sqrt{54} + \sqrt{50})^2 + (\sqrt{54} - \sqrt{50})^2}{4} = \frac{54 + 50 + 54 - 50}{4} = \frac{108}{4} = 27$$

$$c = \sqrt{27} = 3\sqrt{3}$$

$$\text{w/ } a+b = \sqrt{547}$$

$$a = x$$

$$b = \sqrt{547} - x$$

$$\frac{x}{\sin 75^\circ} = \frac{\sqrt{547} - x}{\sin 15^\circ} \quad \text{--- c}$$

$$\text{P/ } a-b = \sqrt{501}$$

$$\frac{x}{\sin 120^\circ \cos 45^\circ - \sin 45^\circ \cos 120^\circ} = \frac{\sqrt{547} - x}{\sin 45^\circ \cos 30^\circ - \cos 30^\circ \sin 80^\circ} \quad \text{--- c}$$

$$\frac{x}{\frac{\sqrt{27}}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right)} = \frac{\sqrt{547} - x}{\frac{\sqrt{27}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right)}$$

$$\frac{x}{\frac{\sqrt{27}}{2} \cdot \frac{\sqrt{3}+1}{2}} = \frac{\sqrt{547} - x}{\frac{\sqrt{27}}{2} \cdot \frac{\sqrt{3}-1}{2}}$$

$$x(\sqrt{3}-1) = (\sqrt{3}+1)(\sqrt{547}-x)$$

$$\sqrt{3}x - x = 3 \cdot 547 - \sqrt{3}x + \sqrt{547} - x$$

$$2\sqrt{3}x = 162$$

$$x = \frac{\sqrt{3}}{81}$$

$$c = \frac{\sqrt{3}}{81} \cdot \frac{4}{\sqrt{27}(\sqrt{3}+1)} = \frac{4\sqrt{3}}{\sqrt{27}(\sqrt{3}+1)}$$

$$= \frac{\sqrt{3}}{81} \cdot \frac{\sqrt{27}(\sqrt{3}-1)}{4(\sqrt{3}-1)} = \frac{\sqrt{67}(\sqrt{3}-1)}{81}$$

$${}^aP_1 = \sqrt{a} = {}^aP_1 = \sqrt{a}$$

$${}^aP_2 = \frac{a(a-1)}{2} = \frac{a(a-1)}{2}$$

$${}^aP_3 = \frac{a(a-1)(a-2)}{6} = \frac{a(a-1)(a-2)}{6}$$

$$= ({}^aP_1) + ({}^aP_2) + ({}^aP_3) = \frac{a(a-1)(a-2)}{6} + \frac{a(a-1)}{2} + \sqrt{a}$$

$$= \left(\frac{a(a-1)(a-2)}{6} + \frac{a(a-1)}{2} + \sqrt{a} \right)$$

$$= \frac{a(a-1)(a-2)}{6} + \frac{a(a-1)}{2} + \sqrt{a}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}$$

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2/4/2014

$$u) \frac{\sin 47^\circ + \sin 13^\circ}{\cos 62^\circ + \cos 28^\circ} = \sqrt{0.5}$$

$$\frac{2 \sin 30^\circ \cos 14^\circ}{2 \cos 45^\circ \cos 14^\circ} = \frac{\sin 30^\circ}{\sin 45^\circ} = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}}$$

$$p) \frac{\sin 41^\circ - \sin 19^\circ}{\cos 34^\circ - \cos 56^\circ} = \sqrt{1.5}$$

$$\frac{2 \sin 22^\circ \cos 30^\circ}{-2 \sin 45^\circ \sin 22^\circ} = \frac{2 \sin 22^\circ \cos 30^\circ}{-2 \sin 22^\circ \sin 45^\circ} = \frac{\sqrt{3}}{2} \cdot \frac{2}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$$

$$q) \sin 42^\circ + \cos 24^\circ - \cos 12^\circ - \sin 6^\circ = 0.5$$

$$(\sin 42^\circ - \cos 12^\circ) + (\cos 24^\circ - \sin 6^\circ) =$$

$$= (\cos 48^\circ - \cos 12^\circ) + (\cos 24^\circ - \cos 84^\circ) =$$

$$= -2 \sin 30^\circ \sin 18^\circ + 2 \sin 51^\circ \sin 60^\circ =$$

$$= -2 \cdot \frac{1}{2} \sin 18^\circ - 2 \cdot \frac{\sqrt{3}}{2} \sin 51^\circ =$$

$$= -\sin 18^\circ - \sqrt{3} \sin 51^\circ$$

Задача 14.5

$$\text{в)} \frac{\sin(\alpha + 30^\circ) - \cos(\alpha + 60^\circ)}{\sin(\alpha + 60^\circ) - \cos(\alpha + 30^\circ)} = \sqrt{3}$$

$$\sin(\alpha + 30^\circ)$$

Р. упр. - 2 ред. гл. - 4

$$\left. \begin{array}{l} \cos \alpha \cos \beta \\ \sin \alpha \sin \beta \\ \sin \alpha \cos \beta \end{array} \right\} \begin{array}{l} \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \end{array}$$

$$\cos \alpha \cos \beta = \frac{1}{2} \left(\cos(\alpha - \beta) + \cos(\alpha + \beta) \right)$$

$$\cos \alpha \sin \beta = \frac{1}{2} \left(\cos(\alpha - \beta) - \cos(\alpha + \beta) \right)$$

$$\sin \alpha \cos \beta = \frac{1}{2} \left(\sin(\alpha + \beta) + \sin(\alpha - \beta) \right)$$

$$u_2. \quad \alpha + \beta = x$$

$$\alpha - \beta = y$$

$$\alpha = \frac{x+y}{2}$$

$$\beta = \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cdot \cos \frac{x+y}{2}$$

$$\log x \pm \log y = \frac{\sin x \pm \sin y}{\cos x \cos y}$$

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \cdot \left(\frac{a}{\sqrt{a^2 + b^2}} \cdot \sin x + \frac{b}{\sqrt{a^2 + b^2}} \cos x \right)$$

$$\left(\frac{a}{\sqrt{a^2 + b^2}} \right)^2 + \left(\frac{b}{\sqrt{a^2 + b^2}} \right)^2 = 1$$

ϕ - auxiliary angle

$$\frac{a}{\sqrt{a^2 + b^2}} = \sin \phi$$

$$\frac{b}{\sqrt{a^2 + b^2}} = \cos \phi$$

$$\sqrt{a^2 + b^2} \cos(x - \phi)$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \cdot \sin \frac{x-y}{2}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cdot \cos \frac{x+y}{2}$$

$$\lg x \pm \lg y = \frac{\sin x \pm \sin y}{\cos x \cos y}$$

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \cdot \left(\frac{a}{\sqrt{a^2 + b^2}} \cdot \sin x + \frac{b}{\sqrt{a^2 + b^2}} \cdot \cos x \right)$$

$$\left(\frac{a}{\sqrt{a^2 + b^2}} \right)^2 + \left(\frac{b}{\sqrt{a^2 + b^2}} \right)^2 = 1$$

φ - остаточно

$$\frac{a}{\sqrt{a^2 + b^2}} = \sin \varphi$$

$$\frac{b}{\sqrt{a^2 + b^2}} = \cos \varphi$$

$$\sqrt{a^2 + b^2} \cos(x - \varphi)$$

Ques 137

$$9) \sin 3x, 5^\circ$$

Ques 140.

$$\begin{aligned} 9) 1 - \cos 4x + \cos 5x - \cos 9x &= \\ &= (1 - \cos 9x) + (\cos 5x - \cos 4x) = \\ &= (1 - \cos 9x) + (-2 \sin \frac{4.5}{2} \sin \frac{x}{2}) = \end{aligned}$$

$$= 2(1 - \cos 9x) - 2 \sin \frac{9x}{2} \sin \frac{x}{2}$$

$$= 2 \sin^2 \frac{9x}{2} - 2 \sin \frac{9x}{2} \sin \frac{x}{2}$$

$$= 2 \sin \frac{9x}{2} \left(\sin \frac{9x}{2} - \sin \frac{x}{2} \right) =$$

$$= 2 \sin \frac{9x}{2} \cdot 2 \sin 2x \cos 5x = 4 \sin 4.5 \sin 4.5 \cos 2.5x$$

Ques 151

$$A) (\sin \alpha + \sin \beta)^2 + (\cos \alpha + \cos \beta)^2 = 4 \cos^2 \frac{\alpha - \beta}{2}$$

$$\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta =$$

$$= 2 + 2 \sin \alpha \cos (\alpha - \beta) - 2 \cos (\alpha + \beta)$$

$$= 2 + 2 (\sin \alpha \sin \beta + \cos \alpha \cos \beta) = 2 + 2 \cos (\alpha - \beta)$$

$$= 2 (1 + \cos (\alpha - \beta)) = 4 \cos^2 \frac{\alpha - \beta}{2}$$

$$7) \frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x$$

$$\frac{2 \sin 3x \cos 2x + \sin 3x}{2 \cos 3x \cos 2x + \cos 3x}$$

$$\frac{\sin 3x (2 \cos 2x + 1)}{\cos 3x (2 \cos 2x + 1)}$$

$$= \tan 3x$$

$$159) \tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$$

$$\frac{\sin 3x}{\cos 3x} - \frac{\sin 2x}{\cos 2x} - \frac{\sin x}{\cos x} =$$

$$\frac{(\sin 3x \cos x - \cos 3x \sin x)}{\cos 3x \cos x}$$

$$\frac{\sin(3x - x)}{\cos 3x \cos x} = \frac{\sin 2x}{\cos x}$$

$$= \frac{\sin x (\cos x - \cos 3x \cos 2x)}{\cos 3x \cos 2x \cos x}$$

$$= \tan x \cdot \frac{(\cos 3x - 2x)}{1}$$

$$\frac{\sin x}{\cos 3x \cos 2x} - \frac{\sin x}{\cos x} = \frac{\sin x \cos x - \sin x \cos 3x \cos 2x}{\cos x \cos 2x \cos 3x}$$

$$= \frac{\sin x (\cos x - \cos 3x \cos 2x)}{\cos x \cos 2x \cos 3x}$$

$$= \frac{\sin x \cdot \cos x - \cos (3x+2x) - \cos (3x-2x)}{\cos 2x \cos 3x}$$

$$= \frac{\sin 2x \cdot \cos x - \cos 5x - \cos x}{\cos 2x \cos 3x}$$

161

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$A+B+C = 180^\circ$$

$$C = 180^\circ - (A+B)$$

$$\sin A + \sin B + \sin (180^\circ - (A+B)) =$$

$$= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} + \sin (A+B) =$$

$$= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} + \frac{2 \sin \frac{A+B}{2} \cos \frac{A+B}{2}}{2} =$$

$$= 2 \sin \frac{A+B}{2} \left(\cos \frac{A-B}{2} + \cos \frac{A+B}{2} \right) =$$

$$= 4 \sin \frac{(180^\circ - C)}{2} \left(\cos \frac{A}{2} \cos \frac{B}{2} + 4 \sin \left(90^\circ - \frac{C}{2} \right) \cos \frac{A}{2} \cos \frac{B}{2} \right)$$

$$= 4 \cos \frac{C}{2} \cos \frac{A}{2} \cos \frac{B}{2}$$

162

$$e) \frac{\cos(x+60^\circ) - \cos(x-60^\circ)}{\cos(x+60^\circ) + \cos(x-60^\circ)} z$$

$$z \frac{\cos x \cos 60^\circ + \sin x \sin 60^\circ - \cos x \cos 60^\circ + \sin x \sin 60^\circ}{\cos x \cos 60^\circ + \sin x \sin 60^\circ + \cos x \cos 60^\circ - \sin x \sin 60^\circ}$$

$$z = \frac{2 \sin x \sin 120^\circ}{2 \cos x \sin 120^\circ} z = \operatorname{tg} x \operatorname{tg} 120^\circ z$$

$$z = \operatorname{tg} x \operatorname{tg} 60^\circ z = \sqrt{3} \operatorname{tg} x$$

Zupad 163

$$A) \cos \frac{\pi}{5} + \cos \frac{3\pi}{5} z \frac{1}{2} \quad \frac{4\pi}{5} z \frac{4 \cdot 180}{5} z$$

$$2 \cos \frac{4\pi}{5} \cos \frac{2\pi}{5} z \frac{1}{2}$$

$$z \quad 4 \cdot 36 = 144$$

$$z \cdot 2 \cos \frac{4\pi}{5} \cos \frac{1}{2} \cdot \frac{2\pi}{5} z$$

$$z \cdot 2 \cos \frac{4\pi}{5} \cdot 2 \left(2 \cos^2 \frac{2\pi}{5} - 1 \right) \cos \frac{2\pi}{5} z$$

$$2 \cos \frac{4\pi}{5}$$

163

$$\cos \frac{\pi}{5} + \cos \frac{3\pi}{5} = 2 \cos \frac{2\pi}{5} \cos \frac{\pi}{5}$$

$$= 2 \times \frac{2\pi}{5} \left(\cos \frac{2\pi}{5} \cdot \frac{\sin \frac{2\pi}{5}}{\sin \frac{2\pi}{5}} - \frac{\sin \frac{4\pi}{5}}{\sin \frac{2\pi}{5}} \cdot \cos \frac{\pi}{5} \right)$$

$$= 2 \frac{\sin \frac{2\pi}{5} \cdot \cos \frac{2\pi}{5}}{2 \sin \frac{\pi}{5}} - \frac{1 \times \sin \frac{4\pi}{5}}{2 \sin \frac{\pi}{5}}$$

$$= \frac{\sin \left(\pi - \frac{\pi}{5} \right)}{2 \sin \frac{\pi}{5}} - \frac{\sin \frac{\pi}{5}}{2 \sin \frac{\pi}{5}} = \frac{1}{2} - \frac{1}{2}$$

165.

$$9) \sin 19^\circ + \sin 25^\circ + \sin 31^\circ + 24 \sin 25^\circ \cos 33^\circ \cos 27^\circ$$

$$= (\sin 19^\circ + \sin 31^\circ) + \sin 25^\circ =$$

$$= 2 \sin 25^\circ \cos 6^\circ + \sin 25^\circ \cdot 2 \sin 25^\circ \left(2 \cos 6^\circ + \frac{1}{2} \right) =$$

$$= 2 \sin 25^\circ (\cos 6^\circ + \cos 60^\circ) =$$

$$= 2 \sin 25^\circ \cos 33^\circ \cos 14^\circ$$

$$= \underline{\underline{-138 - 171}}$$

$$535 - 554$$

Задача 146.

$$u) \left(\frac{2}{1 + \operatorname{tg} x} + \operatorname{tg} 2x \right) \left(\cos^2 x - \frac{1}{2} \right) = \cos^2 x$$

Задача 144.

$$u) \frac{\sin 47^\circ + \sin 13^\circ}{\cos 62^\circ + \cos 28^\circ} = \sqrt{0,5}$$

$$\frac{2 \sin 30^\circ \cos 17^\circ}{2 \cos 45^\circ \cos 17^\circ} = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{2} = \sqrt{\frac{1}{2}}$$

Задача 145.

$$u) \frac{\sin(x+30^\circ) - \cos(x+60^\circ)}{\sin(x+60^\circ) - \cos(x+30^\circ)} = \sqrt{3}$$

$$(\sin x \cos 30^\circ + \cos x \sin 30^\circ -)$$

$$\frac{\cos(x+60^\circ) - \cos(x+60^\circ)}{\sin(x+60^\circ) - \sin(x+60^\circ)} = \sqrt{3}$$

$$= \frac{2 \sin^2(x+60^\circ)}{2 \sin(x+60^\circ) \cos(x+60^\circ)} = \operatorname{tg}(x+60^\circ) =$$

✓

165

$$iii) 16 \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = 1$$

$$8 \cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{8 \sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{\sin 20^\circ}$$

$$= \frac{4 \sin 40^\circ \cos 40^\circ \cos 80^\circ}{\sin 20^\circ} = \frac{2 \sin 80^\circ \cos 80^\circ}{\sin 20^\circ}$$

$$= \frac{2 \sin 160^\circ}{\sin 20^\circ} = \frac{2 \sin 20^\circ}{\sin 20^\circ} = 2 \cdot 1$$

166.

$$ii) \left(\frac{2}{1 + \tan x} + \tan 2x \right) \left(\cos^2 x - \frac{1}{2} \right) = \cos^2 x$$

$$\left(\frac{2}{1 + \tan x} + \tan 2x \right) \left(\cos^2 x - \frac{1}{2} \right) =$$

$$= \left(\frac{2 + \tan 2x + \tan x \tan 2x}{1 + \tan x} \right) \left(\cos^2 x - \frac{1}{2} \right) =$$

$$= \left(\frac{2}{1 + \tan x} + \frac{2 \tan x}{1 - \tan^2 x} \right) \left(\cos^2 x - \frac{1}{2} \right) =$$

$$= \left(\frac{2(1 - \tan x) + 2 \tan x}{(1 + \tan x)(1 - \tan x)} \right) \left(\cos^2 x - \frac{1}{2} \right) =$$

$$= \frac{1}{1 + \tan^2 x} \cdot \cos 2x = \frac{\cos 2x}{1 - \sin^2 x}$$

$$c) \operatorname{tg} x + 2 \operatorname{tg} 2x + 2 \frac{(1 - \operatorname{tg}^2 2x)}{\operatorname{tg} 2x} = 0 \operatorname{tg} x$$

$$- \operatorname{tg} x + \frac{2(2 \operatorname{tg} x)}{1 - \operatorname{tg}^2 x} + \frac{2(1 - \operatorname{tg}^2 x)(1 + \operatorname{tg}^2 x)}{\operatorname{tg} 2x} =$$

$$= \operatorname{tg} x + \frac{4 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} + \frac{2(1 - \operatorname{tg}^2 x)(1 + \operatorname{tg}^2 x)}{2 \operatorname{tg} 2x} =$$

$$= \operatorname{tg} x + \frac{2 \operatorname{tg}^2 2x + 2 - 2 \operatorname{tg}^2 2x}{\operatorname{tg} 2x} =$$

$$= \operatorname{tg} x + \frac{2}{\operatorname{tg} 2x} = \operatorname{tg} x + \frac{2 - 2 \operatorname{tg}^2 x}{2 \operatorname{tg} x} =$$

$$= \frac{\operatorname{tg}^2 x + 1 - \operatorname{tg}^2 x}{\operatorname{tg} x} = \frac{1}{\operatorname{tg} x} = 0 \operatorname{tg} x$$

167)
up) $\frac{\operatorname{tg}(x + \pi) - \operatorname{tg} x - \operatorname{tg} \pi}{\operatorname{tg} x \operatorname{tg}(x + \pi)} = \operatorname{tg} \pi$

$$= \frac{\operatorname{tg} x + \operatorname{tg} \pi}{1 - \operatorname{tg} x \operatorname{tg} \pi} = \operatorname{tg} x - \operatorname{tg} \pi$$

$$\frac{\operatorname{tg} x (\operatorname{tg} x + \operatorname{tg} \pi)}{1 - \operatorname{tg} x \operatorname{tg} \pi}$$

$$z = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta - \operatorname{tg} \alpha + \operatorname{tg} \alpha + \operatorname{tg} \alpha + \operatorname{tg} \alpha - \operatorname{tg} \beta + \operatorname{tg} \beta + \operatorname{tg} \beta + \operatorname{tg} \beta}{\operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg} \alpha + \operatorname{tg} \beta} z$$

$$z = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{tg} \alpha + \operatorname{tg} \beta} z$$

$$z = \frac{\operatorname{tg}(45^\circ + \alpha) - \operatorname{tg}(45^\circ - \alpha)}{\operatorname{tg}(45^\circ + \alpha) + \operatorname{tg}(45^\circ - \alpha)} z$$

$$z = \frac{\operatorname{tg} 45^\circ + \operatorname{tg} \alpha - \operatorname{tg} 45^\circ - \operatorname{tg} \alpha}{1 - \operatorname{tg} 45^\circ \operatorname{tg} \alpha + 1 + \operatorname{tg} 45^\circ \operatorname{tg} \alpha} z$$

z

z = \operatorname{tg} \alpha

$$z = \frac{\operatorname{tg} \frac{\pi}{12} + \operatorname{ctg} \frac{\pi}{12}}{\operatorname{tg} \frac{\pi}{8} + \operatorname{ctg} \frac{\pi}{8}} z = \frac{\left(-\operatorname{tg} \frac{\pi}{12} + \frac{1}{\operatorname{tg} \frac{\pi}{12}} \right)}{\left(\operatorname{tg} \frac{\pi}{8} + \frac{1}{\operatorname{tg} \frac{\pi}{8}} \right)} z = \frac{2 \left(\sin \frac{\pi}{12} + \cos \frac{\pi}{12} \right)}{2 \left(\sin \frac{\pi}{8} + \cos \frac{\pi}{8} \right)} z = \frac{2 \cos \frac{\pi}{12} \sin \frac{\pi}{12}}{2 \cos \frac{\pi}{8} \sin \frac{\pi}{8}} z = \frac{\cos \frac{\pi}{12}}{\cos \frac{\pi}{8}} z = \frac{\cos \frac{\pi}{12}}{\cos \frac{\pi}{8}} z$$

$$z = \frac{1 - \cos \frac{\pi}{6} + \cos \frac{\pi}{6} + 1}{\sin \frac{\pi}{6}} \cdot \frac{\sin \frac{\pi}{4}}{1 - \cos \frac{\pi}{4} + \cos \frac{\pi}{4} + 1} z$$

$$z = \frac{\sin \frac{\pi}{4}}{\sin \frac{\pi}{6}} z = \frac{\sqrt{2}}{2} z = \sqrt{2} z$$

$$f) \frac{\operatorname{ctg} \frac{5\pi}{12} - \operatorname{tg} \frac{6\pi}{12}}{\operatorname{ctg} \frac{2\pi}{8} - \operatorname{tg} \frac{3\pi}{8}} = \sqrt{3}$$

$$\frac{1}{\operatorname{tg} \frac{5\pi}{12}} - \operatorname{tg} \frac{5\pi}{12} \cdot \frac{1 - \operatorname{tg}^2 \frac{5\pi}{12}}{2} = \frac{\operatorname{ctg} \frac{5\pi}{12}}{1 - \operatorname{tg}^2 \frac{3\pi}{8}} = \frac{\operatorname{tg} \frac{3\pi}{8}}{\operatorname{tg} \frac{3\pi}{8}}$$

$$= \frac{2 \operatorname{ctg} \frac{5\pi}{6}}{2 \operatorname{ctg} \frac{3\pi}{4}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

170.

$$u) \frac{2 \cos 50^\circ + \cos 70^\circ}{\sqrt{3} \sin 70^\circ} = 1$$

$$\frac{2 \cos 50^\circ + \cos 70^\circ}{\sqrt{3} \sin 70^\circ}$$

$$\frac{2 \cos(60^\circ - 10^\circ) + \cos(60^\circ + 10^\circ)}{\sqrt{3} \sin 70^\circ}$$

$$= 2 \left(\frac{1}{2} \cos 10^\circ + \right.$$

$$\left. \cos(60^\circ - 10^\circ) \right) = 2 \left(\frac{1}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \sin 10^\circ \right) + \cos \frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ = \sqrt{3} \sin 70^\circ$$

$$= \frac{\cos 10^\circ + \sqrt{3} \sin 10^\circ + \frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ}{\sqrt{3} \left(\frac{\sqrt{3}}{2} \cos 10^\circ + \frac{1}{2} \sin 10^\circ \right)} = 1$$

$$= \frac{\frac{3}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \sin 10^\circ}{\frac{3}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \cos 10^\circ} = 1$$

II b2.
$$\frac{\cos 50^\circ + 2 \cos 60^\circ \cos 10^\circ}{\sqrt{3} \sin 70^\circ} = \frac{\cos 50^\circ + \cos 10^\circ}{\sqrt{3} \sin 70^\circ} =$$

$$= \frac{2 \cos 30^\circ \cos 20^\circ}{\sqrt{3} \sin 70^\circ} = \frac{\sqrt{3} \cos 20^\circ}{\sin 70^\circ} = \frac{\cos (90^\circ - 70^\circ)}{\sin 70^\circ} =$$

$$= \frac{\sin 70^\circ}{\sin 70^\circ} = 1$$

P)
$$\frac{2 \sin 70^\circ - \sqrt{3} \cos 10^\circ}{\sin 10^\circ} = 1$$

$$\frac{2 \sin 70^\circ - \sqrt{3} \cos 10^\circ}{\sin 10^\circ} = \frac{\sqrt{3} \cos 10^\circ + \sin 10^\circ - \sqrt{3} \cos 10^\circ}{\sin 10^\circ} = 1$$

9)
$$\frac{1}{\sin 50^\circ} + \frac{\sqrt{3}}{\cos 50^\circ} = 4$$

$$\frac{\cos 50^\circ + \sqrt{3} \sin 50^\circ}{\sin 50^\circ \cos 50^\circ} = \frac{\cos 50^\circ + \frac{\sqrt{3}}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ}{2}$$

$$= 2 \left(\frac{1}{2} \cos 50^\circ + \frac{\sqrt{3}}{2} \sin 50^\circ \right) = 4 \frac{\sin 80^\circ}{\sin 100^\circ} = 4$$

$$= 4 \frac{\cos 10^\circ}{\cos 10^\circ} = 4$$

$$2) \frac{\sqrt{37}}{\cos 70^\circ} - \frac{1}{\sin 70^\circ} = 4$$

$$\frac{\sqrt{37} \sin 70^\circ - \cos 70^\circ}{\cos 70^\circ \sin 70^\circ} = \frac{2 \left(\frac{\sqrt{37}}{2} \sin 70^\circ - \frac{1}{2} \cos 70^\circ \right)}{\frac{1}{2} \sin 140^\circ}$$

$$\frac{\cos 30^\circ \sin (30 - 70)^\circ}{\sin 140^\circ}$$

$$z = 4 \frac{\sin 40^\circ}{\sin 140^\circ} \quad z = 4 \frac{\sin 40^\circ}{\sin 40^\circ} \quad z = 4$$

Эксперт 8 $\frac{m}{L}$ 4 8 $\frac{L}{m}$ п.м. 1/2.

26. 02. 2006p.

2/урд 137

$$\text{мы } \sin 34,5^\circ \cdot \sin 7,5^\circ = \frac{1}{2} \left(\cos (34,5 - 7,5) + \cos (34,5 + 7,5) \right) =$$

$$= \frac{1}{2} \left(\cos 30^\circ \cos + \cos 45^\circ \right) = \frac{1}{2} \left(\frac{\sqrt{37}}{2} + \frac{\sqrt{27}}{2} \right) = \frac{\sqrt{37} + \sqrt{27}}{4}$$

$$P) \cos 34,5^\circ \cos 7,5^\circ = \frac{1}{2} \left(\cos 45^\circ + \cos 30^\circ \right) = \frac{1}{2} \left(\frac{\sqrt{27}}{2} + \frac{\sqrt{37}}{2} \right) =$$

$$= \frac{\sqrt{27} + \sqrt{37}}{4}$$

$$\begin{aligned}
 4) \sin 34,5^\circ \cos 4,5^\circ &= \frac{1}{2} (\sin 45^\circ + \sin 30^\circ) = \frac{1}{2} \left(\frac{\sqrt{2}}{2} + \frac{1}{2} \right) = \\
 &= \frac{\sqrt{2}+1}{4}
 \end{aligned}$$

$$7) \sin 75^\circ \sin 15^\circ = \frac{1}{2} (\cos 60^\circ - \cos 90^\circ) = \frac{1}{2} \left(\frac{1}{2} - 0 \right) = \frac{1}{4}$$

2/урок 138

$$\text{из)} \sin 75^\circ \cos 15^\circ = \frac{1}{2} (\sin 90^\circ + \sin 60^\circ) = \frac{1}{2} \left(1 + \frac{\sqrt{3}}{2} \right) = \frac{2+\sqrt{3}}{4}$$

$$\text{Р)} \cos 64,5^\circ \sin 22,5^\circ = \frac{1}{2} (\sin 90^\circ - \sin 45^\circ) = \frac{1}{2} \left(1 - \frac{\sqrt{2}}{2} \right) = \frac{2-\sqrt{2}}{4}$$

$$4) \cos 64,5^\circ \cos 22,5^\circ = \frac{1}{2} (\cos 90^\circ + \cos 45^\circ) = \frac{\sqrt{2}}{4}$$

$$7) \cos 94,5^\circ \cos 34,5^\circ = \frac{1}{2} (\cos 60^\circ + \cos 135^\circ) = \frac{1}{2} \left(\frac{1}{2} - \frac{\sqrt{2}}{2} \right) = \frac{1-\sqrt{2}}{4}$$

2/урок 139

$$\text{из)} \cos \frac{5\pi}{12} \cos \frac{\pi}{4} = \frac{1}{2} \left(\cos \frac{\pi}{6} + \cos \frac{2\pi}{3} \right) = \frac{1}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) = \frac{\sqrt{3}-1}{4}$$

$$\text{Р)} \cos \frac{5\pi}{12} \sin \frac{\pi}{4} = \frac{1}{2} \left(\sin \frac{2\pi}{3} - \sin \frac{\pi}{6} \right) = \frac{1}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) = \frac{\sqrt{3}-1}{4}$$

$$4) \sin \frac{\pi}{12} \cos \frac{\pi}{4} = \frac{1}{2} \left(\sin \frac{\pi}{3} + \sin \frac{\pi}{6} \right) = \frac{1}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) = \frac{\sqrt{3}+1}{4}$$

$$7) \sin \frac{\pi}{12} \sin \frac{\pi}{4} = \frac{1}{2} \left(\cos \frac{\pi}{3} - \cos \frac{\pi}{2} \right) = \frac{1}{2} \left(\frac{1}{2} - 0 \right) = \frac{1}{4}$$

2/4/2014

$$u) \sin x + \sin 5x = 2 \sin 3x \cos 2x =$$

$$= 2 \sin 3x \cos 2x$$

$$p) \sin 8x - \sin 2x = 2 \sin 3x \cos 5x$$

$$q) \cos 3y - \cos y = -2 \sin 2y \sin 2y$$

$$r) \cos 2x + \cos 4x = 2 \cos 3x \cos x$$

2/4/2014

$$u) \sin x + \cos 3x = \cos \left(\frac{\pi}{2} - x \right) + \cos 3x = 2 \cos \left(\frac{\frac{\pi}{2} + 2x}{2} \right) \cos \left(\frac{\frac{\pi}{2} - 4x}{2} \right) =$$

$$= 2 \cos \left(\frac{\pi}{4} + x \right) \cos \left(\frac{\pi}{4} - 2x \right)$$

$$p) \sin 2x - \cos 2y = \cos \left(\frac{\pi}{2} - 2x \right) - \cos 2y = 2 \sin \left(\frac{\frac{\pi}{2} - 2x + 2y}{2} \right) \sin \left(\frac{\frac{\pi}{2} - 2x - 2y}{2} \right) =$$

$$= -2 \sin \left(\frac{\pi}{4} - x + y \right) \sin \left(\frac{\pi}{4} - x - y \right)$$

$$q) \cos y + \sin 3y = \sin \left(\frac{\pi}{2} - y \right) + \sin 3y = 2 \sin \left(\frac{\frac{\pi}{2} - y + 3y}{2} \right) \cos \left(\frac{\frac{\pi}{2} - y - 3y}{2} \right) =$$

$$= 2 \sin \left(\frac{\pi}{4} + y \right) \cos \left(\frac{\pi}{4} - 2y \right)$$

$$r) \cos 4x - \sin 2y = \cos 4x - \cos \left(\frac{\pi}{2} - 2y \right) =$$

$$= -2 \sin \left(\frac{\frac{\pi}{2} - 2y + 4x}{2} \right) \sin \left(\frac{4x - \frac{\pi}{2} + 2y}{2} \right) =$$

$$z = -2 \sin\left(\frac{\pi}{4} - y + 2x\right) \sin\left(2x + y - \frac{\pi}{4}\right)$$

group 142

$$\text{w/ } \sin 12^\circ + \sin 24^\circ = 2 \sin 18^\circ \cos 6^\circ$$

$$A) \cos \frac{11\pi}{12} + \cos \frac{3\pi}{4} = 2 \cos \frac{7\pi}{8} \cos \frac{\pi}{12} \quad \text{or } 19 - \sqrt{3} \cos \frac{\pi}{12}$$

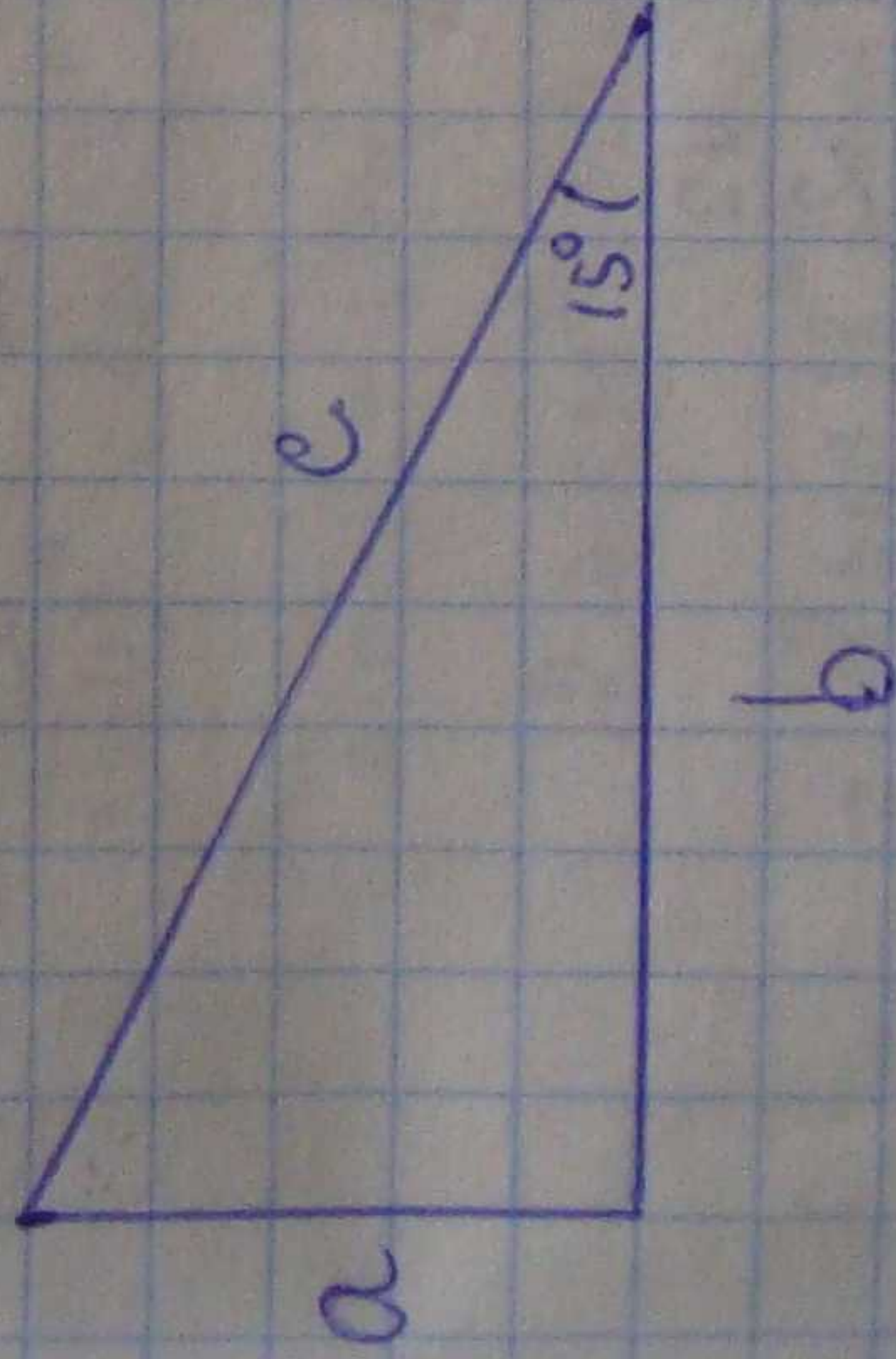
$$B) \cos \frac{\pi}{6} - \cos \frac{7\pi}{12} = 2 \sin\left(-\frac{5\pi}{24}\right) \sin \frac{3\pi}{8} = 2 \sin \frac{3\pi}{8} \sin \frac{5\pi}{24}$$

$$C) \sin \frac{2\pi}{5} + \sin \frac{\pi}{5} = 2 \sin \frac{3\pi}{10} \cos \frac{\pi}{10}$$

group 143

$$a + b = \sqrt{54}$$

$$\text{or } a - b = \sqrt{54}$$



$$a = c \sin 15^\circ$$

$$b = c \cos 15^\circ$$

$$c (\sin 15^\circ + \cos 15^\circ) = \sqrt{54}$$

$$\sin 15^\circ + \cos 15^\circ = \cos \frac{\pi}{4} (90^\circ - 15^\circ) + \cos 15^\circ =$$

$$= \cos 75^\circ + \cos 15^\circ = 2 \cos 45^\circ \cos 30^\circ = 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$$

$$c = \frac{\sqrt{54}}{\sin 15^\circ + \cos 15^\circ} = \frac{\sqrt{54}}{\sqrt{6}} \cdot 2 = 6$$

f) $a - b = \sqrt{50}$

$$a = c \cdot \sin 15^\circ$$

$$b = c \cdot \cos 15^\circ$$

$$c(\sin 15^\circ + \cos 15^\circ) = \sqrt{50}$$

$$\sin 15^\circ - \cos 15^\circ = \cos 45^\circ - \cos 15^\circ = (-2 \sin 45^\circ \sin 30^\circ =)$$

$$(-1) \left(2 \sin 15^\circ - \sin \left(\frac{\pi}{2} - 15^\circ \right) \right)$$

$$= 2 \sin 15^\circ - \sin 45^\circ = 2 \sin 45^\circ \cos 30^\circ = 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$$

$$\left(c = \frac{\sqrt{50}}{\sqrt{6}} \cdot 2 = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3} \right)$$

$$= 2 \sin 30^\circ \cos 45^\circ = -2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$

$$c = \frac{\sqrt{50}}{\sqrt{2}} \cdot 2 = 10$$

2/4/2018 144

$$\text{u} \frac{\sin 47^\circ + \sin 13^\circ}{\cos 62^\circ + \cos 28^\circ} = \sqrt{0.5}$$

$$\frac{2 \sin 30^\circ \cos 47^\circ}{2 \cos 45^\circ \cos 17^\circ} = \frac{1}{\sqrt{2}} = \sqrt{\frac{1}{2}} = \sqrt{0.5}$$

$$F) \frac{\sin 41^\circ - \sin 19^\circ}{\cos 34^\circ - \cos 56^\circ} = \sqrt{1.5}$$

$$\frac{2 \sin 11^\circ \cos 30^\circ}{2 \sin 45^\circ \sin(-11^\circ)} = \frac{\cos 30^\circ}{\sin 45^\circ} = \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}} = \sqrt{1.5}$$

$$9) \sin 42^\circ + \cos 24^\circ - \cos 12^\circ - \sin 6^\circ = 0.5$$

$$(\sin 42^\circ - \sin 6^\circ) + (\cos 24^\circ - \cos 12^\circ) =$$

$$= 2 \sin 18^\circ \cos 24^\circ + 2 \sin 18^\circ \sin 6^\circ =$$

$$= 2 \sin 18^\circ (\cos 24^\circ - \sin 6^\circ) =$$

$$= 2 \sin 18^\circ (\cos 24^\circ - \sin(30^\circ - 24^\circ)) =$$

$$= 2 \sin 18^\circ (\cos 24^\circ - \sin 30^\circ \cos 24^\circ + \cos 30^\circ \sin 24^\circ) =$$

$$= 2 \sin 18^\circ (\cos 24^\circ - \frac{1}{2} \cos 24^\circ + \frac{\sqrt{3}}{2} \sin 24^\circ) =$$

$$= 2 \sin 18^\circ (\frac{1}{2} \cos 24^\circ + \frac{\sqrt{3}}{2} \sin 24^\circ)$$

$$= 2 \sin 18^\circ (\cos 24^\circ - \cos(84^\circ)) = 2 \sin 18^\circ (-2 \sin 54^\circ \sin(-30^\circ)) =$$

$$= 2 \sin 18^\circ \cdot 2 \sin 54^\circ \cdot \frac{1}{2} = 2 \sin 18^\circ \sin 54^\circ$$

Expand 145

$$\text{we } \frac{\sin(\alpha+30^\circ) - \cos(\alpha+60^\circ)}{\sin(\alpha+60^\circ) - \cos(\alpha+30^\circ)} \cdot \sqrt{3}$$

$$\frac{(\sin(\alpha+30^\circ) - \sin(\alpha+30^\circ))}{\sin \alpha \cos 30^\circ + \sin 30^\circ \cos \alpha - \cos \alpha \cos 60^\circ + \sin \alpha \sin 60^\circ}$$

$$\frac{\sin \alpha \cos 60^\circ + \cos \alpha \sin 60^\circ - \cos \alpha \cos 30^\circ + \sin \alpha \sin 30^\circ}{\frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha - \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha}$$

$$= \frac{\frac{1}{2} \sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha - \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha}{\frac{\sqrt{3}}{2} \sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha} = \frac{\frac{1}{2} \sin \alpha}{\frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha}$$

$$= \frac{\sqrt{3} \sin \alpha}{\sin \alpha} = \sqrt{3}$$

$$f) \frac{\cos(\alpha - \frac{\pi}{4}) - \sin(\alpha - \frac{\pi}{4})}{\cos(\alpha - \frac{\pi}{3}) - \sin(\alpha - \frac{\pi}{6})} = \sqrt{2}$$

$$\frac{\cos \alpha \cos \frac{\pi}{4} + \sin \alpha \sin \frac{\pi}{4} - \sin \alpha \cos \frac{\pi}{4} + \cos \alpha \sin \frac{\pi}{4}}{\cos \alpha \cos \frac{\pi}{3} + \sin \alpha \sin \frac{\pi}{3} - \sin \alpha \cos \frac{\pi}{6} + \cos \alpha \sin \frac{\pi}{6}}$$

$$= \frac{\frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \sin \alpha - \frac{\sqrt{2}}{2} \sin \alpha + \frac{\sqrt{2}}{2} \cos \alpha}{\frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha - \frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha} =$$

$$= \frac{\sqrt{2} \cos \alpha}{\cos \alpha} = \sqrt{2}$$

q/urp 146

$$\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$$

$$\operatorname{tg} \alpha \operatorname{tg} \beta = \operatorname{tg}(\alpha + \beta) (1 - \operatorname{tg} \alpha \operatorname{tg} \beta) = \operatorname{tg}(\alpha + \beta) \left(1 - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} \right)$$

$$= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \cdot \frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}$$

$$= \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \cdot \frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}$$

$$= \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$$

1) $\operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta}$

$$\operatorname{tg} \alpha - \operatorname{tg} \beta = \operatorname{tg}(\alpha - \beta) (1 + \operatorname{tg} \alpha \operatorname{tg} \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} \cdot \left(1 + \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} \right)$$

$$= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} \cdot \frac{\cos(\alpha - \beta) + \sin \alpha \sin \beta}{\cos \alpha \cos \beta}$$

of up d 147

$$w) \operatorname{tg} x - \operatorname{tg} 3x = z - \frac{\sin 2x}{\cos x \cos 3x} = z - 2 \frac{\sin x \cos x}{\cos x \cos 3x} = z - 2 \frac{\sin x}{\cos 3x}$$

$$f) \operatorname{tg} y + \operatorname{tg} 3x = z \frac{\sin(3x+y)}{\cos 3x \cos y}$$

$$g) \operatorname{tg} \frac{\pi}{12} - \operatorname{tg} \frac{\pi}{3} = z \frac{\sin(\frac{\pi}{12} - \frac{\pi}{3})}{\cos \frac{\pi}{12} \cos \frac{\pi}{3}} = z - \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{12} \cos \frac{\pi}{3}}$$

$$z = \frac{\sqrt{2}}{2} \cdot z \frac{\pi}{\cos \frac{\pi}{12}} = z - \frac{\sqrt{2}}{\cos \frac{\pi}{12}}$$

$$n) \operatorname{tg} \frac{4\pi}{5} - \operatorname{tg} \frac{3\pi}{5} = z \frac{\sin \frac{\pi}{5}}{\cos \frac{4\pi}{5} \cos \frac{3\pi}{5}}$$

$$(z) \frac{\sin \frac{4\pi}{5}}{\cos \frac{4\pi}{5}} \cos \frac{3\pi}{5} = z \operatorname{tg} \frac{4\pi}{5} \cos \frac{3\pi}{5}$$

$$z \frac{\sin \frac{\pi}{5}}{\frac{1}{2} (\cos \frac{7\pi}{5} + \cos \frac{\pi}{5})}$$

$$z \frac{\sin \frac{4\pi}{5}}{\cos \frac{4\pi}{5}} \cdot \frac{1}{\cos \frac{3\pi}{5}} = z \frac{\operatorname{tg} \frac{4\pi}{5}}{\cos \frac{3\pi}{5}}$$

Exmp 148

$$\text{w/ } \frac{1}{2} + \cos x = \cos 60^\circ + \cos x = 2 \cos \frac{60+x}{2} \cos \frac{60-x}{2}$$

$$= 2 \cos \left(30^\circ + \frac{x}{2} \right) \cos \left(30^\circ - \frac{x}{2} \right) = 2 \cos \left(\frac{x}{2} + \frac{\pi}{6} \right) \cos \left(\frac{x}{2} - \frac{\pi}{6} \right)$$

$$f) \frac{\sqrt{3}}{2} + \sin 2x = \sin 2 \cdot 30^\circ + \sin 2x = 2 \sin(30+x) \cos(30-x)$$

$$= 2 \sin \left(x + \frac{\pi}{6} \right) \cos \left(x - \frac{\pi}{6} \right)$$

$$g) 1 - \sin x = (\sin^2 x - \sin x + \cos^2 x) \cdot \frac{(1 - \sin^2 x)(1 + \sin x)}{1 + \sin x}$$

$$= \frac{1 - \sin^2 x}{1 + \sin x} = \frac{\cos^2 x}{1 + \sin x}$$

$$= \sin 90^\circ - \sin x = \sin \frac{\pi}{2} - \sin x = 2 \sin \left(\frac{\pi}{4} - \frac{x}{2} \right) \cos \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

$$h) \frac{1}{2} + \sin 4x = \sin 30 \frac{\pi}{6} + \sin 4x = 2 \sin \left(\frac{\pi}{2} + 2x \right) \cos \left(\frac{\pi}{2} - 2x \right)$$

$$i) \frac{\sqrt{3}}{2} - \sin 2x = \sin \frac{\pi}{3} - \sin 2x = 2 \sin \left(\frac{\pi}{6} - x \right) \cos \left(\frac{\pi}{6} + x \right)$$

$$j) \frac{\sqrt{2}}{2} + \cos x = \cos \frac{\pi}{4} + \cos x = 2 \cos \left(\frac{\pi}{8} + \frac{x}{2} \right) \cos \left(\frac{\pi}{8} - \frac{x}{2} \right)$$

2/sep/149

$$\text{ex) } \sin x + \sin 2x + \sin 3x + \sin 4x =$$

$$= (\sin 3x + \sin x) + (\sin 4x + \sin 2x) =$$

$$= 2 \sin 2x \cos x + 2 \sin 3x \cos x =$$

$$= 2 \cos x (\sin 2x + \sin 3x) = 2 \cos x \cdot 2 \sin \frac{5}{2} x \cos \frac{x}{2} = 4 \cos x \sin \frac{5}{2} x \cos \frac{x}{2}$$

$$\text{p) } \cos x - \cos 3x + \cos 5x - \cos 7x =$$

$$= -2 \sin 2x (\cos 5x - \cos 3x) - (\cos 7x - \cos x) =$$

$$= -2 \sin 2x \sin x + 2 \sin 4x \sin 3x =$$

$$= 2 \sin 4x (\sin 3x - \sin x) = 2 \sin 4x \cdot (2 \sin 2x \cos x) =$$

$$\text{q) } 1 - \cos 4x + \cos 5x - \cos 9x =$$

$$= \cos 0 (\cos 0 - \cos 4x) +$$

$$= (\cos 5x - \cos 4x) - (\cos 9x - \cos 0) =$$

$$= -2 \sin 4,5x \sin 0,5x + 2 \sin 4,5x \sin 4,5x =$$

$$= 2 \sin 4,5x (\sin 4,5x - \sin 0,5x) =$$

$$= 2 \sin \frac{9}{2} x \cdot 2 \sin 2x \cos \frac{5}{2} x =$$

$$= 4 \sin \frac{9}{2} x \sin 2x \cos \frac{5}{2} x$$

051 p 2 km/2

$$u) \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{\sin(\alpha + \beta) - \sin(\alpha - \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta}$$

$$\frac{(tg\alpha + tg\beta) \cos\alpha \cos\beta + (tg\alpha - tg\beta) \cos\alpha \cos\beta}{(tg\alpha + tg\beta) \cos\alpha \cos\beta - (tg\alpha - tg\beta) \cos\alpha \cos\beta} = 2$$

$$\frac{2 \cos \beta \tan \alpha}{2 \cos \beta \tan \beta} = \tan \alpha \cot \beta$$

$$2) \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha - \beta) - \cos(\alpha + \beta)} = \cot \alpha \cot \beta$$

$$\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta + \cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta - \cos \alpha \cos \beta + \sin \alpha \sin \beta}$$

$$z \frac{\cos 2 \cos \beta}{\sin 2 \sin \beta} = 2 \cot \beta \cot \gamma$$

IS/ Pzhm/2

$$\text{imp } \frac{\cos d + \sin d}{\cos d - \sin d} \quad 2 \operatorname{tg} / 45^\circ + 2$$

$$\frac{\cos x + \sin x}{\cos x - \sin x} = \frac{\cos x + \cos(\frac{\pi}{2} - x)}{\cos x - \cos(\frac{\pi}{2} - x)}$$

$$z \frac{2 \cos \frac{\pi}{2} \cos 1}{2 \cos \pi \cos (\lambda - \pi)} = \frac{\cos \pi \cos 1}{\sin \pi \sin \lambda} = -\frac{\cos 1}{\sin \lambda}$$

2 ctg 5 ctg L

$$p) (\sin d + \sin \beta)^2 + (\cos d + \cos \beta)^2 = 4 \cos^2 \frac{d+\beta}{2}$$

$$\left(2 \sin \frac{d+\beta}{2} \cos \frac{d-\beta}{2} \right)^2 + \left(2 \cos \frac{d+\beta}{2} \cos \frac{d-\beta}{2} \right)^2 =$$

$$= \frac{4 \sin^2 \frac{d+\beta}{2} \cos^2 \frac{d-\beta}{2}}{2} + \frac{4 \cos^2 \frac{d+\beta}{2} \cos^2 \frac{d-\beta}{2}}{2}$$

$$= \frac{4 \cos^2 \frac{d-\beta}{2} \left(\sin^2 \frac{d+\beta}{2} + \cos^2 \frac{d+\beta}{2} \right)}{2} =$$

$$= \frac{4 \cos^2 \frac{d-\beta}{2}}{2}$$

$$q) (\sin 2d + \sin 4d)^2 + (\cos 2d + \cos 4d)^2 = 4 \cos^2 d$$

$$\left(2 \sin 3d \cos d \right)^2 + \left(2 \cos 3d \cos d \right)^2 =$$

$$= 4 \sin^2 3d \cos^2 d + 4 \cos^2 3d \cos^2 d =$$

$$= 4 \cos^2 d (\sin^2 3d + \cos^2 3d) =$$

$$= 4 \cos^2 d$$

$$r) \frac{\sin d + \sin 3d + \sin 5d}{\cos d + \cos 3d + \cos 5d} = \operatorname{tg} 3d$$

$$\frac{2 \sin 2d \cos d + \sin 5d}{2 \sin 2d \cos d + \sin 5d}$$

$$\frac{2 \sin 3x \cos 2x + \sin 3x}{2 \cos 3x \cos 2x + \cos 3x} = \frac{\sin 3x (2 \cos 2x + 1)}{\cos 3x (2 \cos 2x + 1)} =$$

$$= \tan 3x$$

Wsp.	Wspereq	Polynom 152
	$x \cdot 45/d$	Area
z.B.S.	$(x-18)/45/d$	60/5
		75p 200p
		kurz $\frac{5}{4}$ bis
		kurz 2145 bis 247

Wird die Spannung für die

Wpue 9000 7 kg

habe die Spannung

für die

$$\frac{5}{4} (x-18) \cdot 45$$

$$60 - 21 - \frac{5}{4} (x-18) \cdot 45 = 60 - 21 - \frac{5}{4} (x-18) \cdot 45$$

Wird die Spannung für die

$$\frac{39 - \frac{5}{4} (x-18)}{x-18} = \frac{60}{x}$$

$$\frac{156 - 5(x-18)}{4(x-18)} = \frac{60}{x}$$

$$156x - 5x^2 + 90x = 240x - 720$$



$$5x^2 - 6x - 4320 = 0$$

$$\frac{ab}{4} = 36 + 8640 = 8676 = 9 + 8600 = 4329 \approx 147^2$$

$$X_{1,2} = \frac{3 \pm 147}{5} \quad \text{--- 2b 1-147}$$

$$x + 18 = 48607$$

$$= 12$$

7-1: 48607

pba.	$x \cdot \frac{1}{5} \cdot 1$	792
clapn	$(x+5) \cdot \frac{1}{5} \cdot 1$	pba 153
		120/s
		pba 153

hinterfragt die p. bei 45/s. hba 4/s



after 45/s. hba 4/s. hba 4/s

hinterfragt die p. bei 45/s. hba 4/s

hinterfragt die p. bei 45/s. hba 4/s

hinterfragt die p. bei 45/s. hba 4/s

hinterfragt die p. bei 45/s. hba 4/s

$$\frac{120 - 0,5(x+s) - 45}{x+s} = \frac{45}{x}$$

$$\frac{75 - 0,5x - 2,5}{x+s} = \frac{45}{x}$$

$$\frac{75}{x+s} - 0,5 = \frac{45}{x}$$

$$\frac{75}{x+s} = \frac{45 + 0,5x}{x}$$

$$75x = (x+s)(45 + 0,5x)$$

$$75x = 45x + 0,5x^2 + 22,5 + 0,5xs$$

$$0,5x^2 - 27,5s + 22,5 = 0$$

$$sx^2 - 27,5s + 22,5 = 0$$

$$x = \frac{27,5 \pm 17,5}{2}$$

$$x_{1/2} = \frac{27,5 \pm 17,5}{2}$$

Result: $\left[\frac{45}{10}, \frac{10}{10} \right]$

$$\begin{array}{l|l} \text{I und II} & \text{P/308} \\ \hline 9 \rightarrow f_{\text{hoh}} \cdot x & 180/5 \\ \text{P/308} & (x+20) \end{array}$$

$$45p = \frac{45}{60} \cdot \frac{9}{15} = \frac{3}{4} \text{ fms}$$

$$\frac{180}{x} = \frac{180}{x+20} \cdot \frac{3}{4}$$

$$\frac{180(x+20-x)}{x(x+20)} = \frac{3}{4}$$

$$180003600 \cdot 5 = 3x^2 + 60x$$

$$x^2 + 20x - 6000 = 0$$

$$\frac{20}{4} \pm \sqrt{\frac{100}{4} + 6000} = 6100$$

$$3x^2 + 60x = 14400$$

$$x^2 + 20x - 4800 = 0$$

$$b = 100 + 4800 = 4900$$

$$x_{1,2} = \frac{-10 \pm 70}{2}$$

$$x + 20 = 80/57d$$

$$P/308 \cdot 60/57d \cdot 80/57d$$

$$n/2 \text{ p/308}$$

$$\cos^4 x - \sin^4 x = \cos 2x$$

$$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos 2x$$

$$P) (\lg x + \text{ctg} x) \sin 2x = 2$$

$$\frac{\lg^2 x + 1}{\lg x} \sin 2x = \lg x + \text{ctg} x$$

$$= \left(\frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} \right) \cdot \frac{2 \sin x \cos x}{\sin x} = \frac{2 \cos x}{\sin x} = 2 \cot x$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{2 \sin x \cos x}{\cos x \sin x} = 2$$

$$9) \frac{\cos^2 x}{1 + \sin x} + \sin x = 1$$

$$\frac{\cos^2 x + \sin x + \sin^2 x}{1 + \sin x} = \frac{1 + \sin x}{1 + \sin x} = 1$$

$$11) \frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$$

$$\frac{1 - \tan^2 x}{\tan x} \cdot \frac{-\tan x}{1 + \tan^2 x} = \frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos 2x$$

$$= \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{1 - \sin^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos^2 x}{1} = \cos^2 x$$

2/урд 156

$$\text{у)} \cos^2(\pi + \alpha) + \cos^2\left(\frac{\pi}{2} + \alpha\right) = 1$$
$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$\text{Р)} \sin\left(\frac{\pi}{2} + \alpha\right) \cdot \sin(\pi - \alpha) = \frac{1}{2} \cos\left(\frac{3\pi}{2} + 2\alpha\right)$$

$$\cos \alpha \cdot \sin \alpha = \frac{1}{2} \sin 2\alpha = \frac{1}{2} \cos\left(\frac{3}{2}\pi - 2\alpha\right)$$

2/урд 157

$$\text{у)} (1 + \operatorname{ctg}^2 \alpha)(1 - \sin^2 \alpha) = \operatorname{ctg}^2 \alpha$$

$$\frac{\cos^2 \alpha + \sin^2 \alpha}{\sin^2 \alpha} \cdot \cos^2 \alpha = \frac{\cos^2 \alpha}{\sin^2 \alpha} = \operatorname{ctg}^2 \alpha$$

$$\text{Р)} (1 + \operatorname{tg}^2 \alpha)(1 - \cos^2 \alpha) = \operatorname{tg}^2 \alpha$$

$$\frac{\cos^2 \alpha + \sin^2 \alpha}{\cos^2 \alpha} \cdot \sin^2 \alpha = \operatorname{tg}^2 \alpha$$

$$\text{г)} \frac{\sin \alpha + \cos \alpha \operatorname{tg} \alpha}{\cos \alpha + \sin \alpha \operatorname{ctg} \alpha} = 2 \operatorname{tg} \alpha$$

$$\frac{2 \cos \alpha \operatorname{tg} \alpha}{2 \sin \alpha \operatorname{ctg} \alpha} = \operatorname{tg} \alpha \cdot \operatorname{tg}^2 \alpha = \operatorname{tg}^3 \alpha$$

$$\frac{2 \sin \alpha}{2 \cos \alpha} = \operatorname{tg} \alpha$$

з/урд 158

$$\text{ур} \frac{1}{1+\operatorname{tg}^2 x} + \frac{1}{1+\operatorname{ctg}^2 x} = \frac{\cos^2 x}{\cos^2 x + \sin^2 x} + \frac{\sin^2 x}{\sin^2 x + \cos^2 x} = 1$$

$$p) \frac{1}{1+\operatorname{tg}^2 x} - \frac{1}{1+\operatorname{ctg}^2 x} = \cos 2x$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

з/урд 159

$$\text{ур} \operatorname{tg} 3x - \operatorname{tg} 2x - \operatorname{tg} x = \operatorname{tg} 3x \operatorname{tg} 2x \operatorname{tg} x$$

$$\operatorname{tg} 3x - \operatorname{tg} 2x - \operatorname{tg} x = \frac{\sin x}{\cos 3x \cos 2x} - \operatorname{tg} x$$

$$= \frac{\sin x}{\cos 3x \cos 2x} - \frac{\sin x}{\cos x} = \frac{\sin x \cos x - \sin x \cos 3x \cos 2x}{\cos 3x \cos 2x \cos x}$$

$$= \operatorname{tg} x \frac{(\cos x - \cos 3x \cos 2x)}{\cos 3x \cos 2x} = \operatorname{tg} x \frac{\frac{1}{2}(\cos 5x + \cos x)}{\cos 3x \cos 2x}$$

$$= \operatorname{tg} x \left(\frac{\cos x}{\cos 3x \cos 2x} - 1 \right) =$$

$$= \operatorname{tg} x \left(\frac{2 \cos x}{2 \cos 3x \cos 2x} - 1 \right) =$$

$$= \operatorname{tg} x \cdot \frac{2 \cos x + \cos x - \cos 5x}{2 \cos 3x \cos 2x} = \frac{\cos x - \cos 5x}{\cos 5x + \cos x} = \frac{2 \sin 3x \cos 2x}{2 \cos 3x \cos 2x}$$

$$\operatorname{tg} 3x - \operatorname{tg} 2x - \operatorname{tg} x =$$

$$= \frac{\sin 3x}{\cos 3x} - \frac{\sin 2x}{\cos 2x} - \operatorname{tg} x = \frac{\sin 3x \cos 2x - \sin 2x \cos 3x}{\cos 3x \cos 2x} - \operatorname{tg} x =$$

$$= \frac{\sin x}{\cos 3x \cos 2x} - \frac{\sin x}{\cos x} =$$

$$= \frac{\sin x (\cos x - \cos 3x \cos 2x)}{\cos x \cos 2x \cos 3x} =$$

$$= \operatorname{tg} x \frac{(\cos x - \frac{1}{2} (\cos 5x + \cos x))}{\cos 2x \cos 3x} =$$

$$= \operatorname{tg} x \frac{(2 \cos x - \cos x - \cos 5x)}{2 \cos 2x \cos 3x} =$$

$$= \operatorname{tg} x \frac{\cos x - \cos 5x}{2 \cos 2x \cos 3x} = \operatorname{tg} x - \frac{2 \sin 3x \sin 2x}{2 \cos 3x \cos 2x} =$$

$$= - \operatorname{tg} 3x \operatorname{tg} 2x \operatorname{tg} x$$

$$A) \cos x \cos 2x \cos 4x \cos 8x \cos 16x = \frac{\sin 32x}{32 \sin x}$$

$$\frac{1}{2} (\cos 3x + \cos x) \frac{1}{2} (\cos 12x + \cos 4x) \frac{1}{2} \cos 16x =$$

$$= \frac{1}{4} (\cos 3x \cos 12x + \cos 3x \cos 4x +$$

उपरा 160

$$\sin^4 x = 2 \sin^2 x \cos^2 x = 4 \sin^2 x \cos^2 x (\cos^2 x - \sin^2 x) =$$

$$= 4 \sin^2 x \cos^2 x - 4 \sin^4 x \cos^2 x$$

$$\cos 4x = \cos^2 2x - \sin^2 2x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) =$$

$$= (\cos^2 x - \sin^2 x - 2 \sin^2 x \cos^2 x) (\cos^2 x + \sin^2 x + 2 \sin^2 x \cos^2 x) =$$

$$= \cos^4 x + \cos^2 x \sin^2 x + 2 \sin^2 x \cos^2 x - \sin^4 x \cos^2 x -$$

$$- 2 \sin^3 x \cos^3 x - 2 \sin^2 x \cos^3 x - 2 \sin^3 x \cos^2 x - 4 \sin^2 x \cos^2 x =$$

$$= \cos^4 x - \sin^4 x - 4 \sin^3 x \cos^2 x - 4 \sin^2 x \cos^3 x =$$

$$= \cos^2 x - \sin^2 x - 4 \sin^3 x \cos^2 x - 4 \sin^2 x \cos^3 x =$$

$$= \cos^2 x (1 - 4 \sin^2 x) - \sin^2 x (1 + 4 \sin^2 x) =$$

$$= \cos^2 x (\cos^2 x - 3 \sin^2 x) - \sin^2 x (\sin^2 x + 3 \cos^2 x)$$

उपरा 161

$$\text{उपरा } \frac{\sin(45^\circ + x) + \sin(45^\circ - x)}{\sin(45^\circ + x) - \sin(45^\circ - x)} = \frac{2 \sin 45^\circ \cos x}{2 \sin x \cos 45^\circ} = \frac{2 \sin 45^\circ \cos x}{2 \sin x \cos 45^\circ}$$

$$P) \frac{\cos(x + 60^\circ) - \cos(x - 60^\circ)}{\cos(x + 60^\circ) + \cos(x - 60^\circ)} = \frac{-2 \sin x \cos 60^\circ}{2 \cos x \sin 60^\circ}$$

$$= -\frac{\sqrt{3}}{3} \cdot \tan x$$

4

891 p.d.m/c

$$w) \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} = -\frac{1}{2}$$

$$\left(\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} \right)^2$$

$$= \cos^2 \frac{2\pi}{7} + \cos^2 \frac{4\pi}{7} + \cos^2 \frac{6\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{6\pi}{7} + 2 \cos \frac{4\pi}{7} \cos \frac{6\pi}{7}$$

$$= \cos^2 \frac{2\pi}{7} + \cos^2 \frac{4\pi}{7} + \cos^2 \frac{6\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{6\pi}{7} + 2 \cos \frac{4\pi}{7} \cos \frac{6\pi}{7}$$

$$= 2 \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} \cos \frac{6\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{4\pi}{7} + \cos^2 \frac{6\pi}{7}$$

$$= \cos \frac{6\pi}{7} \left(2 \cos \frac{2\pi}{7} + 1 \right)$$

*

$$\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} = \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + 1 \right) = \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} \right)$$

$$= \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + 1 \right) = \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + \cos^2 \frac{2\pi}{7} + \sin^2 \frac{2\pi}{7} \right)$$

$$= \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + \cos^2 \frac{2\pi}{7} + 1 - \cos^2 \frac{2\pi}{7} \right)$$

2/4/2017 163

$$\text{w/ } \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} = -\frac{1}{2}$$

$$\left(\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} \right)^2$$

$$= \cos^2 \frac{2\pi}{7} + \cos^2 \frac{4\pi}{7} + \cos^2 \frac{6\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{6\pi}{7} + 2 \cos \frac{4\pi}{7} \cos \frac{6\pi}{7}$$

$$= \cos^2 \frac{2\pi}{7} + \cos^2 \frac{4\pi}{7} + \cos^2 \frac{6\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{6\pi}{7} + 2 \cos \frac{4\pi}{7} \cos \frac{6\pi}{7}$$

$$= 2 \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} + 2 \cos \frac{2\pi}{7} \cos \frac{6\pi}{7} + 2 \cos \frac{4\pi}{7} \cos \frac{6\pi}{7}$$

$$= 2 \cos \frac{6\pi}{7} \left(2 \cos \frac{2\pi}{7} + 1 \right)$$

*

$$\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} = 2 \cos \frac{4\pi}{7} \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}$$

$$= \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + 1 \right) = \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \sin^2 \frac{2\pi}{7} \right)$$

$$= \cos \frac{4\pi}{7} \left(2 \cos \frac{2\pi}{7} + \cos^2 \frac{2\pi}{7} + 1 - \cos^2 \frac{2\pi}{7} \right)$$

$$f) \cos \frac{\pi}{5} + \cos \frac{3\pi}{5} = \frac{1}{2}$$

$$2 \cos \frac{2\pi}{5} \cos \frac{\pi}{5} =$$

2/4/2016

$$0 \leq x \leq \frac{\pi}{2}$$

$$\cos \sqrt{1 + \sin x} + \sqrt{1 - \sin x} = 2 \cos \frac{x}{2}$$

$$\left(\frac{\cos x}{\sqrt{1 - \sin x}} + \frac{\cos x}{\sqrt{1 + \sin x}} \right) = \frac{\cos x}{\sqrt{1 - \sin^2 x}}$$

$$\sqrt{1 + \cos\left(\frac{\pi}{2} - x\right)} + \sqrt{1 - \cos\left(\frac{\pi}{2} - x\right)} =$$

$$= \sqrt{2} \cos\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) =$$

$$= \sqrt{2} \left(\cos\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \right) =$$

$$= \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos \frac{x}{2} - \frac{\sqrt{2}}{2} \sin \frac{x}{2} + \frac{\sqrt{2}}{2} \cos \frac{x}{2} + \frac{\sqrt{2}}{2} \sin \frac{x}{2} \right) =$$

$$= \sqrt{2} \cdot \sqrt{2} \cos \frac{x}{2} = 2 \cos \frac{x}{2}$$

$$P) \sqrt{1+\sin x} - \sqrt{1-\sin x} = 2 \sin \frac{x}{2}$$

$$\sqrt{1+\cos\left(\frac{\pi}{2}-x\right)} - \sqrt{1-\cos\left(\frac{\pi}{2}-x\right)} =$$

$$= \sqrt{2} \cos\left(\frac{\pi}{4} - \frac{x}{2}\right) - \sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) =$$

$$= \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos \frac{x}{2} + \frac{\sqrt{2}}{2} \sin \frac{x}{2} \right) - \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos \frac{x}{2} - \frac{\sqrt{2}}{2} \sin \frac{x}{2} \right) =$$

$$= 2 \sin \frac{x}{2}$$

∴ $\frac{1}{\sin 165}$

$$w) 16 \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = 1$$

$$8 (\cos 60^\circ + \cos 20^\circ) \cdot \frac{1}{2} \cos 80^\circ =$$

$$= 2 \left(2 \cdot \frac{1}{2} \cdot \cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ \right) =$$

$$= 2 (\cos 80^\circ + \cos 100^\circ + \cos 60^\circ) =$$

$$= 2 \left(2 \cos 90^\circ \cos 10^\circ + \frac{1}{2} \right) = 1$$

$$P) \sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ = \cos 4^\circ$$

$$\left(\sin 47^\circ + \sin 61^\circ \right) - \left(\sin 25^\circ + \sin 11^\circ \right) =$$

$$= 2 \sin 7^\circ \cos 54^\circ - 2 \sin 18^\circ \cos 7^\circ$$

$$P) \sqrt{1 + \sin x} - \sqrt{1 - \sin x} = 2 \sin \frac{x}{2}$$

$$\sqrt{1 + \cos\left(\frac{\pi}{2} - x\right)} - \sqrt{1 - \cos\left(\frac{\pi}{2} - x\right)} =$$

$$= \sqrt{2} \cos\left(\frac{\pi}{4} - \frac{x}{2}\right) - \sqrt{2} \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) =$$

$$= \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos \frac{x}{2} + \frac{\sqrt{2}}{2} \sin \frac{x}{2} \right) - \sin \frac{\sqrt{2}}{2} \cos \frac{x}{2} + \frac{\sqrt{2}}{2} \sin \frac{x}{2} =$$

$$= 2 \sin \frac{x}{2}$$

$\therefore$ $\frac{2}{\sqrt{2}} \sin \frac{x}{2}$

$$w) 16 \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = 1$$

$$8 (\cos 60^\circ + \cos 20^\circ) \cdot \frac{1}{2} \cos 80^\circ =$$

$$= 2 \left(\frac{1}{2} \cos 80^\circ + 2 \cos 20^\circ \cos 80^\circ \right) =$$

$$= 2 (\cos 80^\circ + \cos 100^\circ + \cos 60^\circ) =$$

$$= 2 \left(2 \cos 90^\circ \cos 10^\circ + \frac{1}{2} \right) = 1$$

$$p) \sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ = \cos 7^\circ$$

$$\left((\sin 47^\circ + \sin 61^\circ) - (\sin 25^\circ + \sin 11^\circ) \right) =$$

$$= 2 \sin 7^\circ \cos 54^\circ - 2 \sin 18^\circ \cos 7^\circ$$

$$\begin{aligned}
&= (\sin 44^\circ - \sin 11^\circ) + (\sin 61^\circ - \sin 25^\circ) = \\
&= 2 \sin 18^\circ \cos 29^\circ + 2 \sin 18^\circ \cos 43^\circ = \\
&= 2 \sin 18^\circ (\cos 29^\circ + \cos 43^\circ) = 4 \sin 18^\circ \cos 36^\circ \cos 4^\circ = \\
&= (4 \sin 18^\circ (\cos^2 18^\circ - \sin^2 18^\circ) \cos 4^\circ) = 2 (\sin 36^\circ \cos 18^\circ - 2 \sin^3 18^\circ) \\
&= 4 \sin 18^\circ \sin 54^\circ \cos 4^\circ = 2 (\sin 36^\circ + \sin 18^\circ) \cos 4^\circ = \\
&= 2 (2 \sin 18^\circ \cos 18^\circ + \sin 18^\circ) \cos 4^\circ = \\
&= 2 \sin 18^\circ (2 \cos 18^\circ + 1) \cos 4^\circ = 2 \sin 18^\circ (\cos 18^\circ + 2 \cos^2 9^\circ) \cos 4^\circ
\end{aligned}$$

$$\begin{aligned}
\text{Q1) } \sin 19^\circ + \sin 25^\circ + \sin 31^\circ &= 4 \sin 25^\circ \cos 33^\circ \cos 24^\circ \\
2 \sin 25^\circ \cos 6^\circ + \sin 25^\circ &= \sin 25^\circ (2 \cos 6^\circ + 1) = \\
&= \sin 25^\circ (2 \cos (33^\circ - 24^\circ) + 1) = \\
&= \sin 25^\circ (2 (\cos 33^\circ \cos 24^\circ + \sin 33^\circ \sin 24^\circ) + 1) = \\
&= \sin 25^\circ (\cos 60^\circ + \cos 6^\circ - \cos 6^\circ + \cos 60^\circ + 1) = \\
&= \sin 25^\circ \left(\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} \right) = \sin 25^\circ \frac{2 + \sqrt{3}}{2} \\
&= \sin 25^\circ (2 \cos 33^\circ \cos 24^\circ - \cos 6^\circ + \cos 60^\circ + 1) = \\
&= \sin 25^\circ (2 \cos 33^\circ \cos 24^\circ - \cos 6^\circ + 1.5)
\end{aligned}$$

$$\begin{aligned}
\text{Q1) } \sin 16^\circ + \sin 24^\circ + \sin 40^\circ &= 4 \sin 20^\circ \cos 12^\circ \cos 8^\circ \\
2 \sin 20^\circ \cos 4^\circ + \sin 40^\circ &= 2 \sin 20^\circ \cos 4^\circ + 2 \sin 20^\circ \cos 20^\circ = \\
&= 2 \sin 20^\circ (\cos 4^\circ + \cos 20^\circ) = 2 \sin 20^\circ \cdot 2 \cos 12^\circ \cos 8^\circ = \\
&= 4 \sin 20^\circ \cos 12^\circ \cos 8^\circ
\end{aligned}$$

зад 166.

$$u) \left(\frac{2}{1+\operatorname{tg} x} + \operatorname{tg} 2x \right) \left(\cos^2 x - \frac{1}{2} \right) = \cos^2 x$$

$$\left(\frac{2}{1+\operatorname{tg} x} + \frac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x} \right) \left(\cos^2 x - \frac{1}{2} \right) =$$

$$= \frac{2(1-\operatorname{tg} x) + 2\operatorname{tg} x}{1-\operatorname{tg}^2 x} \left(\cos^2 x - \frac{1}{2} \right) =$$

$$= \frac{2 \left(\cos^2 x - \frac{1}{2} \right)}{1-\operatorname{tg}^2 x} = \frac{\cos 2x}{1-\operatorname{tg}^2 x} = \frac{\cos 2x \cdot \cos^2 x}{\cos^2 x}$$

$$= \cos^2 x$$

$$r) \operatorname{tg} x + 2\operatorname{tg} 2x + \frac{2(1-\operatorname{tg}^2 x)}{\operatorname{tg} 2x} = \operatorname{ctg} x$$

$$\left(\operatorname{tg} x + \frac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x} + \frac{2(1-\operatorname{tg} 2x)(1+\operatorname{tg} 2x)}{\operatorname{tg} 2x} \right) =$$

$$= \frac{\operatorname{tg} x \cdot \operatorname{tg} 2x + 2\operatorname{tg}^2 x + 2 - 2\operatorname{tg}^2 2x}{\operatorname{tg} 2x} =$$

$$= \frac{2\operatorname{tg}^2 x}{1-\operatorname{tg}^2 x} + 2\operatorname{tg}^2 x + 2 - 8\operatorname{tg}^2 x = \frac{(1-\operatorname{tg}^2 x)^2}{1-\operatorname{tg}^2 x} = \operatorname{ctg} x$$

$$= \frac{(2tg^2x + 2tg^2x - 2tg^2x)}{2}$$

$$= \frac{2tg^2x(1+tg^2x) + 2tg^2x(1-tg^2x) + 2(1-tg^2x) - 8tg^2x}{2}$$

$$= \frac{2tg^2x + 2tg^2x + 2tg^2x - 2tg^2x + 2 - 2tg^2x - 8tg^2x}{2}$$

$$= \frac{1 - 2tg^2x}{2tg^2x} \cdot \frac{1}{tg^2x} = \frac{1 - 2tg^2x - 4tg^2x}{2tg^2x}$$

$$= \frac{1 - tg^2x + 4tg^2x - 1}{2tg^2x} = \frac{2tg^2x - 2}{2tg^2x} = \frac{tg^2x - 1}{tg^2x}$$

$$= \frac{1}{tg^2x} + \frac{4tg^2x}{1 - tg^2x} + \frac{2(1 - tg^2x)(1 + tg^2x)}{2tg^2x}$$

$$= \frac{1}{tg^2x} + \frac{4tg^2x}{1 - tg^2x} + \frac{2(1 - tg^2x)(1 + tg^2x)(1 - tg^2x)}{2tg^2x}$$

2/sep 167

$$\frac{\frac{\alpha\beta}{1-\alpha\beta} - \frac{\alpha\beta}{1-\alpha\beta}}{\frac{\alpha\beta}{1-\alpha\beta} - \frac{\alpha\beta}{1-\alpha\beta}} = \frac{\alpha\beta}{1-\alpha\beta}$$

$$\frac{\alpha\beta}{1-\alpha\beta} = \frac{\alpha\beta}{1-\alpha\beta}$$

$$\frac{\alpha\beta}{1-\alpha\beta} = \frac{\alpha\beta}{1-\alpha\beta}$$

$$\frac{\alpha\beta}{1-\alpha\beta} = \frac{\alpha\beta}{1-\alpha\beta}$$

$$\frac{\alpha\beta}{1-\alpha\beta} = \frac{\alpha\beta}{1-\alpha\beta}$$

$$\frac{\sin 45^\circ}{\sin 90^\circ} = \frac{\sin 45^\circ}{\sin 90^\circ}$$

2/sep 168

$$\frac{\sin \alpha}{\cos^3 \alpha} = \frac{\sin \alpha}{\cos^3 \alpha}$$

$$\frac{\sin \alpha}{\cos^3 \alpha} = \frac{\sin \alpha}{\cos^3 \alpha}$$

$$z(tg\alpha + 1)(tg^3\alpha + 5)z - tg^3\alpha + tg\alpha + tg^3\alpha + 5$$

$$p)(tg^2\alpha - 2tg\alpha)(ctg\alpha - tg\alpha) = 2tg^2\alpha$$

$$\left(\frac{2tg\alpha}{1-tg^2\alpha} - 2tg\alpha\right)\left(\frac{1-tg^2\alpha}{tg\alpha}\right) = \frac{2tg\alpha - 2tg\alpha + 2tg^3\alpha}{1-tg^2\alpha} \cdot \frac{1-tg^2\alpha}{2tg\alpha}$$

$$= \frac{2tg^3\alpha}{1-tg^2\alpha} \cdot \frac{1-tg^2\alpha}{tg\alpha} = \frac{2tg^3\alpha}{tg\alpha} = 2tg^2\alpha$$

2/упр 169

$$\frac{-tg\frac{\pi}{12} + ctg\frac{\pi}{12}}{-tg\frac{\pi}{8} + ctg\frac{\pi}{8}}$$

уп

$$= \sqrt{2}$$

$$\frac{-tg^2\frac{\pi}{12} + 1}{-tg\frac{\pi}{12}} \cdot \frac{-tg\frac{\pi}{8} + 1}{-tg^2\frac{\pi}{8} + 1} = \frac{1 - \cos\frac{\pi}{6}}{1 + \cos\frac{\pi}{6}} \cdot \frac{1 - \cos\frac{\pi}{6}}{1 + \cos\frac{\pi}{6}} = \frac{1 - \cos\frac{\pi}{6}}{1 + \cos\frac{\pi}{6}}$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}} \cdot \frac{1 + \frac{\sqrt{2}}{2}}{1 - \frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \cdot \frac{2 + \sqrt{2}}{2 - \sqrt{2}}$$

$$g) \sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} = \left(\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} \right)^2 - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} =$$

$$= 1 - 2 \sin^2 \frac{\pi}{8} = 1 - \frac{1}{2} \sin \frac{\pi}{4} = 1 - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{2 - \sqrt{2}}{2}$$

$$h) \sin^4 \frac{\pi}{12} + \cos^4 \frac{\pi}{12} = 1 - 2 \sin^2 \frac{\pi}{12} \cos^2 \frac{\pi}{12} = 1 - \frac{1}{2} \sin^2 \frac{\pi}{6} =$$

$$= 1 - \frac{1}{2} \cdot \frac{1}{4} = \frac{7}{8}$$

2/4/2017

$$u) \frac{2 \cos 50^\circ + \cos 70^\circ}{\sqrt{3} \sin 40^\circ} = 1$$

$$\frac{\cos 50^\circ + 2 \cos 60^\circ \cos 10^\circ}{\sqrt{3} \sin 70^\circ} = \frac{\cos 60^\circ \cos 10^\circ + \sin 60^\circ \sin 10^\circ + 2 \cos 60^\circ \cos 10^\circ}{\sqrt{3} (\sin 60^\circ \cos 10^\circ - \cos 60^\circ \sin 10^\circ)}$$

$$= \frac{3 \cdot \frac{1}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \sin 10^\circ}{\sqrt{3} \cdot \frac{\sqrt{3}}{2} \cos 10^\circ - \frac{1}{2} \sin 10^\circ} = \frac{\sqrt{3} \cdot \frac{3}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \sin 10^\circ}{\sqrt{3} \cos 10^\circ - \frac{1}{2} \sin 10^\circ}$$

$$= \frac{\sqrt{3} \cos 10^\circ + \frac{1}{2} \sin 10^\circ}{\sqrt{3} \cos 10^\circ - \frac{1}{2} \sin 10^\circ}$$

2/cup d 70

$$u) \frac{2 \cos 50^\circ + \cos 70^\circ}{\sqrt{3} \sin 70^\circ} = \frac{2 \cos(60^\circ - 10^\circ) + \cos(60^\circ + 10^\circ)}{\sqrt{3} \sin 70^\circ}$$

$$= \frac{2 \left(\frac{1}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \sin 10^\circ \right) + \frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ}{\sqrt{3} \sin 70^\circ}$$

$$= \frac{\cos 10^\circ + \sqrt{3} \sin 10^\circ + \frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ}{\sqrt{3} (\sin 60^\circ \cos 50^\circ)}$$

$$= \frac{\sqrt{3} \left(\frac{\sqrt{3}}{2} \cos 10^\circ + \frac{1}{2} \sin 10^\circ \right)}{\sqrt{3} \sin 70^\circ}$$

$$= \frac{\frac{3}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \sin 10^\circ}{\frac{3}{2} \cos 10^\circ + \frac{\sqrt{3}}{2} \sin 10^\circ} = 1$$

$$p) \frac{2 \sin 70^\circ - \sqrt{3} \cos 10^\circ}{\sin 10^\circ} = \frac{2 \left(\frac{\sqrt{3}}{2} \cos 10^\circ + \frac{1}{2} \sin 10^\circ \right) - \sqrt{3} \cos 10^\circ}{\sin 10^\circ}$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} = 1$$

$$q) \frac{1}{\sin 50^\circ} + \frac{\sqrt{3}}{\cos 50^\circ} = \frac{\cos 50^\circ + \sqrt{3} \sin 50^\circ}{\sin 50^\circ \cos 50^\circ}$$

$$= \frac{2 \left(\frac{1}{2} \cos 50^\circ + \frac{\sqrt{3}}{2} \sin 50^\circ \right)}{\frac{1}{2} \sin 100^\circ} = \frac{4 (\sin 30^\circ \cos 50^\circ + \cos 30^\circ \sin 50^\circ)}{\sin 100^\circ}$$

$$= \frac{4 \sin 80^\circ}{\sin 100^\circ} = \frac{4 \sin(180^\circ - 100^\circ)}{\sin 100^\circ} = 4$$

$$7) \frac{\sqrt{3}}{\cos 70^\circ} - \frac{1}{\sin 70^\circ} = \frac{\sqrt{3} \sin 70^\circ - \cos 70^\circ}{\cos 70^\circ \sin 70^\circ} = \frac{2(\cos 30^\circ \sin 70^\circ - \sin 30^\circ \cos 70^\circ)}{\frac{1}{2} \sin 140^\circ} = \frac{4 \sin 40^\circ}{\sin 140^\circ} = \frac{4 \sin 40^\circ}{\sin 140^\circ} = 4$$

$$\text{we} \quad x + \frac{1}{x^2 - 4} = \frac{x(x^2 - 4)}{x^2 - 4} = \frac{1}{(x-2)(x+2)} + x$$

p.w.f. $x \neq \pm 2$

$$P) \frac{x}{x^2 - 5x + 6}$$

$$x^2 - 5x + 6 = 0$$

$$\Delta = 25 - 24 = 1$$

$$x_{1,2} = \frac{5 \pm 1}{2} = \frac{3}{2}$$

$$x^2 - 5x + 6 = (x-2)(x-3) \Rightarrow \text{p.w.f. } x \neq 2; x \neq 3$$

2/4/2014

$$4) \sin 42^\circ + \cos 24^\circ - \cos 12^\circ - \sin 6^\circ = 0.5$$

$$(\sin 42^\circ - \sin 6^\circ)$$

$$= 2 \sin 18^\circ \cos 24^\circ - 2 \sin 6^\circ \sin 18^\circ =$$

$$= 2 \sin 18^\circ (\cos 24^\circ - \sin 6^\circ) =$$

$$= 2 \sin 18^\circ (\sin 66^\circ - \sin 6^\circ) =$$

$$= 2 \sin 18^\circ \cdot 2 \sin 30^\circ \cos 36^\circ =$$

$$= 2 \sin 18^\circ \cos 36^\circ =$$

$$= 2 \cos 18^\circ \sin 18^\circ \cos 36^\circ = \frac{\sin 36^\circ \cos 36^\circ}{\cos 18^\circ} =$$

$$= \frac{\sin 72^\circ}{2 \cos 18^\circ} = \frac{\sin 72^\circ}{2} = \frac{1}{2} = 0.5$$

Page 151

$$\text{we } \frac{\cos x + \sin x}{\cos x - \sin x} = \tan(45^\circ + x)$$

$$\left(\frac{\cos x + \sin x}{\cos x - \sin x} \right)^2 =$$

$$\tan^2(45^\circ + x) = \frac{\sin(45^\circ + x)}{\cos(45^\circ + x)} = \frac{\sin 45^\circ \cos x + \cos 45^\circ \sin x}{\cos 45^\circ \cos x - \sin 45^\circ \sin x} =$$

$$= \frac{\frac{\sqrt{2}}{2} (\cos x + \sin x)}{\frac{\sqrt{2}}{2} (\cos x - \sin x)} =$$

Quest 161

$$P) \quad \frac{\tan A}{2} - \frac{\tan B}{2} + \frac{\tan B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$$

$$\frac{\tan A}{2} - \frac{\tan B}{2} + \frac{\tan B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} =$$

$$= \frac{\tan A}{2} \tan \frac{B}{2} + \tan \frac{C}{2} \left(\frac{\tan B}{2} + \tan \frac{A}{2} \right) =$$

$$= \frac{\tan A}{2} \tan \frac{B}{2} + \tan \left(90 - \frac{A+B}{2} \right) \left(\frac{\tan B}{2} + \tan \frac{A}{2} \right) =$$

$$= \frac{\tan A}{2} \tan \frac{B}{2} + \cot \left(\frac{A+B}{2} \right) \left(\frac{\tan B}{2} + \tan \frac{A}{2} \right) =$$

$$= \frac{\tan \frac{A}{2} \tan \frac{B}{2} + 1 - \frac{\tan A}{2} \tan \frac{B}{2}}{\frac{\tan A}{2} + \tan \frac{B}{2}} \left(\frac{\tan B}{2} + \tan \frac{A}{2} \right) =$$

$$= 1$$

$$Q) \quad \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\tan A + \tan B + \tan(A+B) = \tan A + \tan B + \frac{\tan A + \tan B}{1 - \tan A \tan B} =$$

$$= \frac{(1 - \tan A \tan B)(\tan A + \tan B) + (\tan A + \tan B)}{1 - \tan A \tan B}$$

$$= (\tan A + \tan B)(2 - \tan A \tan B)$$

$$\operatorname{tg} A + \operatorname{tg} B + \operatorname{tg} C =$$

$$\frac{\sin(A+B)}{\cos A \cos B} - \operatorname{tg}(A+B) =$$

$$= \frac{\sin(A+B)}{\cos A \cos B} - \frac{\sin(A+B)}{\cos(A+B)} =$$

$$= \frac{\sin(A+B)(\cos(A+B) - \cos A \cos B)}{\cos A \cos B \cos(A+B)}$$

$$= \frac{\sin(A+B) \sin C}{2} (-\cos C)$$

163.

$$\text{мы } \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} = -\frac{1}{2}$$

$$\left(\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \right)$$

$$\frac{\cos \frac{2\pi}{7} \sin \frac{\pi}{7} + \cos \frac{4\pi}{7} \sin \frac{\pi}{7} + \cos \frac{6\pi}{7} \sin \frac{\pi}{7}}{\sin \frac{\pi}{7}} =$$

$$= \frac{1}{2} \left(\cancel{\sin \frac{3\pi}{7}} + \sin \frac{\pi}{7} + \sin \frac{\pi}{7} + \cancel{\sin \frac{5\pi}{7}} - \cancel{\sin \frac{3\pi}{7}} + \sin \pi + \sin \frac{5\pi}{7} \right) =$$

$$= \frac{1}{2}$$

$$= \frac{\sin \pi - \sin \frac{\pi}{7}}{2 \sin \frac{\pi}{7}} = \frac{0 - \frac{\sin \pi}{2}}{2 \sin \frac{\pi}{7}} = -\frac{1}{2}$$

185.

2/ up simplification.

$$\sin 160^\circ \cos 70^\circ - \cos 200^\circ \sin 70^\circ - \cos 225^\circ \sin 315^\circ = \frac{\tan 55^\circ \tan 235^\circ}{2}$$

$$\frac{\sin 20^\circ \cos 70^\circ - \cos 20^\circ \sin 70^\circ + \cos 45^\circ \sin 45^\circ}{2} = \tan 55^\circ \tan 235^\circ$$

$$\frac{-\sin 90^\circ + \sin 50^\circ - \sin 50^\circ - \cos 45^\circ \sin 45^\circ}{2}$$

$$\frac{-\left(\sin 50^\circ + \frac{1}{2}\right)}{2} = \tan 55^\circ \tan 235^\circ$$

2/ up 535

$$w) \left(\left(\frac{1+\cos x}{\sin x} \right)^2 + 1 \right); \frac{1+\cos x}{\sin^2 x} =$$

$$= \frac{(1+\cos x)^2 + \sin^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1+\cos x} =$$

$$= \frac{1+2\cos x+1}{1+\cos x} = \frac{2(1+\cos x)}{1+\cos x} = 2$$

$$P) \frac{1+\tan x + \tan^2 x}{1+\tan x + \tan^2 x} = \frac{1+\tan x + \tan^2 x}{1+\frac{1}{\tan x} + \frac{1}{\tan^2 x}} = \frac{(1+\tan x + \tan^2 x) \cdot \tan^2 x}{\tan^2 x + \tan x + 1} =$$

$$= \tan^2 x$$

$$9) \frac{\operatorname{tg} x}{1 - \operatorname{tg}^2 x}, \frac{\operatorname{ctg}^2 x - 1}{\operatorname{ctg} x} =$$

$$= \frac{-\operatorname{tg} x}{1 - \operatorname{tg}^2 x} \cdot \frac{1 - \operatorname{tg}^2 x}{\operatorname{tg} x} = 1$$

$$11) \frac{1}{1 + \operatorname{tg}^2 x} + \frac{1}{1 + \operatorname{ctg}^2 x} = \frac{1}{1 + \operatorname{tg}^2 x} + \frac{\operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x} =$$

$$= \frac{1 + \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x} = 1$$

ответ 536

$$12) \operatorname{tg} 225^\circ \cdot \cos 330^\circ \cdot \operatorname{ctg} 120^\circ =$$

$$= \left(\frac{\sin 225^\circ}{\cos 225^\circ} \cdot \cos 330^\circ \cdot \frac{\cos 120^\circ}{\sin 120^\circ} \right) \cdot$$

$$= \operatorname{tg} (180^\circ + 45^\circ) \cdot \cos (360^\circ - 30^\circ) \cdot \operatorname{ctg} (90^\circ + 30^\circ) =$$

$$= \operatorname{tg} 45^\circ \cdot \cos 30^\circ \cdot \operatorname{ctg} (-\operatorname{tg} 30^\circ) =$$

$$= -\operatorname{tg} 45^\circ \cdot \sin 30^\circ = -1 \cdot \frac{1}{2} = -\frac{1}{2}$$

$$P) \operatorname{ctg} \frac{5\pi}{3} \sin \frac{3\pi}{4} - \operatorname{tg} \frac{5\pi}{6} \cos \frac{4\pi}{3} =$$

$$= (\operatorname{ctg} \frac{2\pi}{3} \cdot \sin) - \operatorname{ctg} \frac{\pi}{3} \cdot \sin \frac{\pi}{4} \cdot (-\operatorname{tg} \frac{\pi}{6}) \cdot (-\sin \frac{\pi}{6}) =$$

$$= -\frac{\sqrt{3}}{3} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{3} \cdot \frac{1}{2} = -\frac{\sqrt{2}}{12}$$

$$q) \operatorname{tg} 1^\circ \cdot \operatorname{tg} 3^\circ \cdot \operatorname{tg} 5^\circ \dots \therefore \operatorname{tg} 89^\circ$$

дупд 534.

$$\sin \alpha; \cos \alpha; \operatorname{tg} \alpha; \operatorname{ctg} \alpha \quad 0 < \alpha < \frac{\pi}{6}$$

~~cos~~ не

$$\cos \alpha > \sin \alpha \quad \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \quad \operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\operatorname{tg} \alpha < \operatorname{ctg} \alpha \quad \operatorname{tg} \alpha < \sin \alpha < \cos \alpha$$

$$\operatorname{ctg} \alpha < \cos \alpha$$

$$\cos \alpha \operatorname{ctg} \alpha > \sin \alpha \operatorname{tg} \alpha$$

$$\text{On } \alpha = 15^\circ = 45^\circ - 30^\circ$$

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) =$$

$$= \frac{\sqrt{2}(\sqrt{3}+1)}{4}$$

$$\sin(45^\circ - 30^\circ) = \sin \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) = 0$$

$$\operatorname{ctg}(45^\circ - 30^\circ) = \frac{\cos(45^\circ - 30^\circ)}{\sin(45^\circ - 30^\circ)} = \frac{1}{0} \quad \text{н.в.}$$

Задача 538

$$\text{мы } 0,5 - \sin \frac{7\pi}{8} = 0,5 - \sin \frac{\pi}{8} = \sin 30^\circ - \sin 22,5^\circ > 0$$

$$P) \frac{\sqrt{3}}{2} - \cos \frac{3\pi}{12} = \frac{\sqrt{3}}{2} - \cos \frac{\pi}{4} = \cos \frac{\pi}{6} - \cos \frac{\pi}{4} < 0$$

$$g) \sin 1 - \sin 2 = \sin \frac{1}{3,14} - \sin \frac{2}{3,14} = \sin \frac{1}{3,14} - \sin \frac{1}{1,57} < 0$$

Задача 539

$$\sin \alpha + \cos \alpha = \frac{4}{3}; \sin 2\alpha = ?$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = (\sin \alpha + \cos \alpha)^2 - (\sin^2 \alpha + \cos^2 \alpha) =$$

$$= (\sin \alpha + \cos \alpha)^2 - 1 = \frac{16}{9} - 1 = \frac{7}{9}$$

Задача 540

$$\sin 2\alpha = \frac{2}{3}; \quad 0 < \alpha < \frac{\pi}{2}$$

$$(\sin \alpha + \cos \alpha)^2 = \sin^2 \alpha + \cos^2 \alpha + 2 \sin \alpha \cos \alpha$$

$$\text{мы } \sin \alpha + \cos \alpha = \sqrt{1 + \sin 2\alpha} = \sqrt{1 + \frac{2}{3}} = \sqrt{\frac{5}{3}}$$

$$P) \sin^3 \alpha + \cos^3 \alpha = (\sin \alpha + \cos \alpha)(1 - \sin \alpha \cos \alpha) =$$

$$= \sqrt{\frac{5}{3}} \left(1 - \frac{1}{2} \sin 2\alpha\right) =$$

$$= \sqrt{\frac{5}{3}} \cdot \frac{2}{3} = \frac{2}{3} \cdot \sqrt{\frac{5}{3}}$$

$$\begin{aligned} \text{q) } \sin^4 \alpha + \cos^4 \alpha &= (\sin^2 \alpha + \cos^2 \alpha)^2 - 2\sin^2 \alpha \cos^2 \alpha \\ &= 1 - \frac{1}{2} \sin^2 2\alpha = 1 - \frac{1}{2} \cdot \frac{4}{9} = 1 - \frac{4}{18} = \frac{14}{18} = \frac{7}{9} \end{aligned}$$

2/2 up d 541

$$\sin \alpha + \cos \alpha > 1$$

$$\sin \alpha + \cos \alpha = \sqrt{1 + \sin 2\alpha} > 1$$

$$\sin 2\alpha \geq \sin \alpha \cos \alpha$$

$$\sin 2\alpha \sin \alpha > 0; \cos \alpha > 0 \Rightarrow \sin 2\alpha > 0$$

2/2 up d 542

$$\text{u) } \sin 25^\circ + \cos 15^\circ > 1$$

$$\sin 25^\circ + \cos 15^\circ = \sin(40^\circ +$$

$$= \sin(45^\circ - 20^\circ) + \cos 15^\circ = \frac{\sqrt{2}}{2} \cos 20^\circ + \frac{\sqrt{2}}{2} \sin 20^\circ + \cos 15^\circ$$

$$= \frac{\sqrt{2}}{2} (\cos 20^\circ - \sin 20^\circ) + \cos 15^\circ = \frac{\sqrt{2}}{2} (\cos 60^\circ -$$

$$= -\sin 25^\circ + \cos 15^\circ = \sin 10^\circ \cos 15^\circ + \cos 10^\circ \sin 15^\circ + \cos 15^\circ =$$

$$= \cos 15^\circ (\sin 10^\circ + 1) + \cos 10^\circ \sin 15^\circ$$

$$\sin 25^\circ + \cos 15^\circ = \sin 25^\circ + \sin 75^\circ = \sin 25^\circ + \frac{\sqrt{27}}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) =$$

$$\approx \sin 25^\circ + \frac{\sqrt{67 + \sqrt{27}}}{4} = \sin 25^\circ + \frac{\sqrt{27}}{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) =$$

$$\frac{\sqrt{6} + \sqrt{2}}{4} \sqrt{\frac{8 + 2\sqrt{12}}{4}} = \frac{2 + 4\sqrt{3}}{4}$$

$$= \frac{1}{2} \left(\sqrt{8 + 4\sqrt{3}} \right) = \frac{1}{2} \sqrt{1}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4} \sqrt{\frac{8 + 2\sqrt{12}}{4}} = \frac{2 + 4\sqrt{3}}{4}$$

$$= \frac{\sqrt{2} + \sqrt{3}}{2}$$

$$\frac{\sqrt{2}}{2} (\cos 10^\circ - \sin 10^\circ) + \frac{1}{2} (\sqrt{2 + \sqrt{3}})$$

$\frac{\sqrt{2}}{2}$

of upnd 542

$$\text{iii) } \sin 25^\circ + \cos 15^\circ > 1$$

$$\sin 25^\circ + \cos 15^\circ = \sin 10^\circ \cos 15^\circ + \cos 10^\circ \sin 15^\circ + \cos 15^\circ =$$

$$= \cos 15^\circ (\sin 10^\circ + 1) + \cos 10^\circ \sin 15^\circ =$$

$$= \frac{\sqrt{2}(\sqrt{3}+1)}{2} \left(\sin 10^\circ + 1 \right) + \cos 10^\circ \cdot \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) =$$

$$= \frac{\sqrt{2}(\sqrt{3}+1)}{2} (\sin 10^\circ + 1) + \frac{\sqrt{2}(\sqrt{3}-1)}{2} \cos 10^\circ$$

$$(u) \sin 25^\circ + \cos 15^\circ$$

z/urp d 553

$$u) \frac{2 \sin \alpha \cos \beta - \sin(\alpha + \beta)}{\cos(\alpha + \beta) + 2 \sin \alpha \sin \beta} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta}$$

$$= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \operatorname{tg}(\alpha - \beta)$$

$$p) \operatorname{tg} x \cdot \operatorname{tg}(x + 60^\circ) \cdot \operatorname{tg}(x - 60^\circ) =$$

$$= \operatorname{tg} x \cdot \frac{\operatorname{tg} x + \sqrt{3}}{1 - \sqrt{3} \operatorname{tg} x} \cdot \frac{\operatorname{tg} x - \sqrt{3}}{1 + \sqrt{3} \operatorname{tg} x}$$

$$= \operatorname{tg} x \cdot \frac{\operatorname{tg}^2 x - 3}{1 - 3 \operatorname{tg}^2 x}$$

z/urp d 542

$$u) \text{ туг, нт, } \sin 25^\circ + \cos 15^\circ > 1$$

$$\sin 25^\circ + \cos 25^\circ = \sqrt{1 + 2 \sin 25^\circ \cos 15^\circ}$$

$$\left. \begin{array}{l} \sin 25^\circ > 0 \\ \cos 15^\circ > 0 \end{array} \right\} \Rightarrow \sqrt{1 + 2 \sin 25^\circ \cos 15^\circ} > 1$$

$$f) (\sin 32^\circ + \cos)$$

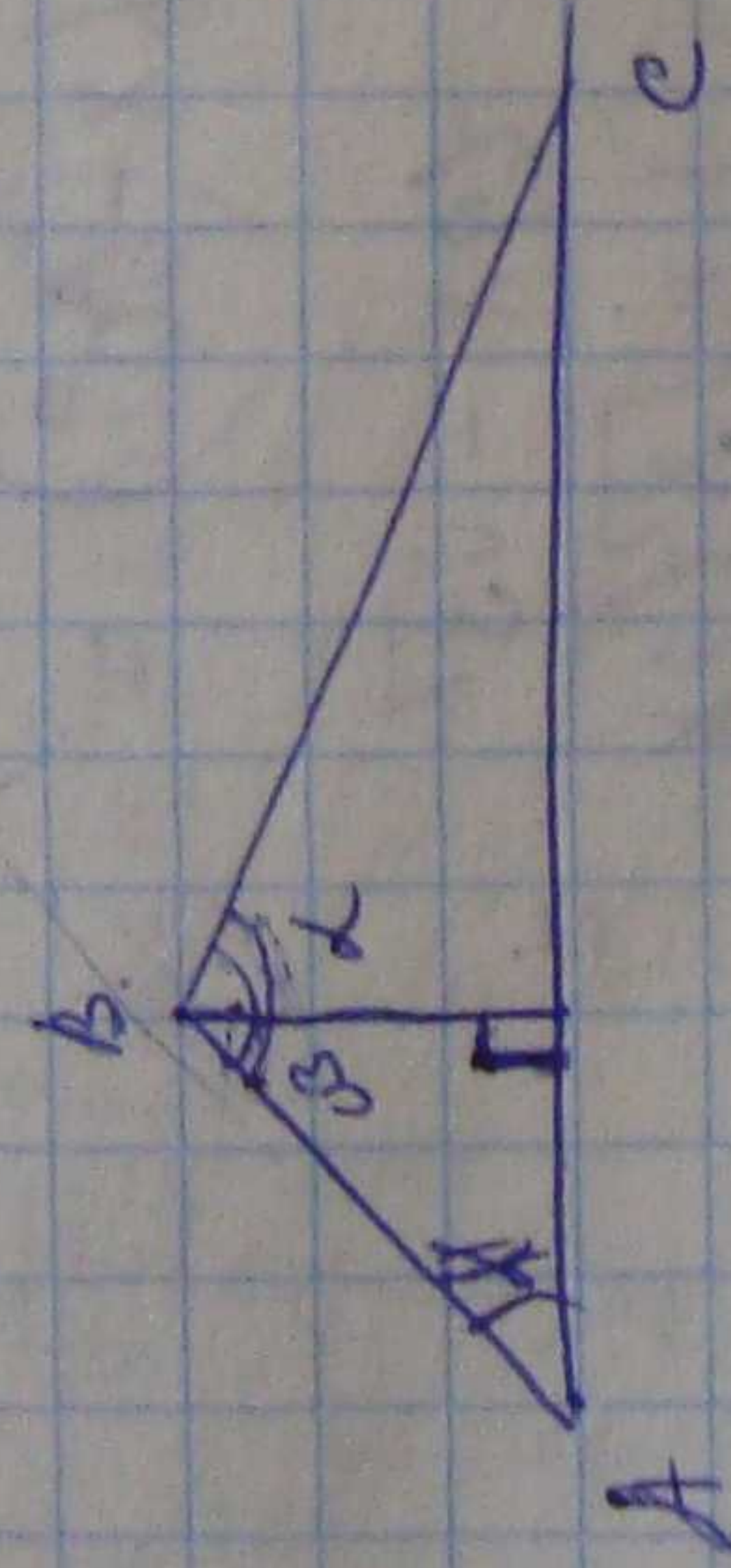
$$\sin 31^\circ + \sin 61^\circ > 1$$

$$\sin 31^\circ + \sin 61^\circ = \sin 31^\circ + \cos 29^\circ = \sqrt{1 + 2\sin 31^\circ \cos 29^\circ} > 1$$

$$\text{für } \sin 31^\circ > 0$$

$$\cos 29^\circ > 0$$

2. Aufg. 543



$$\tan A = \frac{4}{3}$$

$$\tan B = 7$$

$$\text{also, nr } C = 45^\circ$$

$$\tan A = \frac{4}{3} \Rightarrow \cos A = \sqrt{\frac{1}{1 + \tan^2 A}} = \sqrt{\frac{1}{1 + \frac{16}{9}}} = \frac{3}{5}$$

$$\Rightarrow \sin B = \cos A = \frac{3}{5}$$

$$\cos B = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan B = \cos B \Rightarrow \cos B = \sqrt{\frac{1}{1 + \tan^2 B}} = \sqrt{\frac{1}{1 + 50}}$$

$$\tan B = \sqrt{\frac{1}{\cos^2 B} - 1} = \sqrt{\frac{25}{16} - 1} = \frac{3}{4}$$

$$\tan A = \tan B = \tan C = 7 - \frac{3}{4} = \frac{25}{4}$$

$$\cos A = \sqrt{\frac{1}{1 + \tan^2 A}} = \sqrt{\frac{1}{1 + \frac{25^2}{16}}} = \sqrt{\frac{16}{641}}$$

$$= \frac{4\sqrt{641}}{641}$$

1641

$$\sin C = \cos A = \frac{\sqrt{641}}{641}$$

2/10/2019

$$\cos A = \frac{\sqrt{5}}{3}$$

$$\cos 2A = 2\cos^2 A - 1 = 2 \cdot \frac{5}{9} - 1 = \frac{1}{9}$$

$$P) \cos 4A = 2\cos^2 2A - 1 = 2 \left(\frac{1}{9} \right)^2 - 1 =$$

$$= 2 \left(\frac{1}{81} \right) - 1 = 8 \cdot \left(\frac{25}{81} \right) - 1 =$$

$$= 2 \left(\frac{25}{81} \right) - 1 = 2 \left(\frac{4 \cos^4 A + 1 + 4 \cos^2 A}{81} \right) - 1 =$$

$$= 8 \cos^4 A - 8 \cos^2 A + 1 = 8 \left(\cos^4 A + \cos^2 A + 1 \right) + 1 =$$

$$= 8 \cdot \left(\frac{25}{81} - \frac{5}{9} \right) + 1 = 8 \cdot \left(-\frac{20}{81} \right) + 1 = -\frac{80}{81} + 1 =$$

$$= 1 - \frac{80}{81} = \frac{1}{81}$$

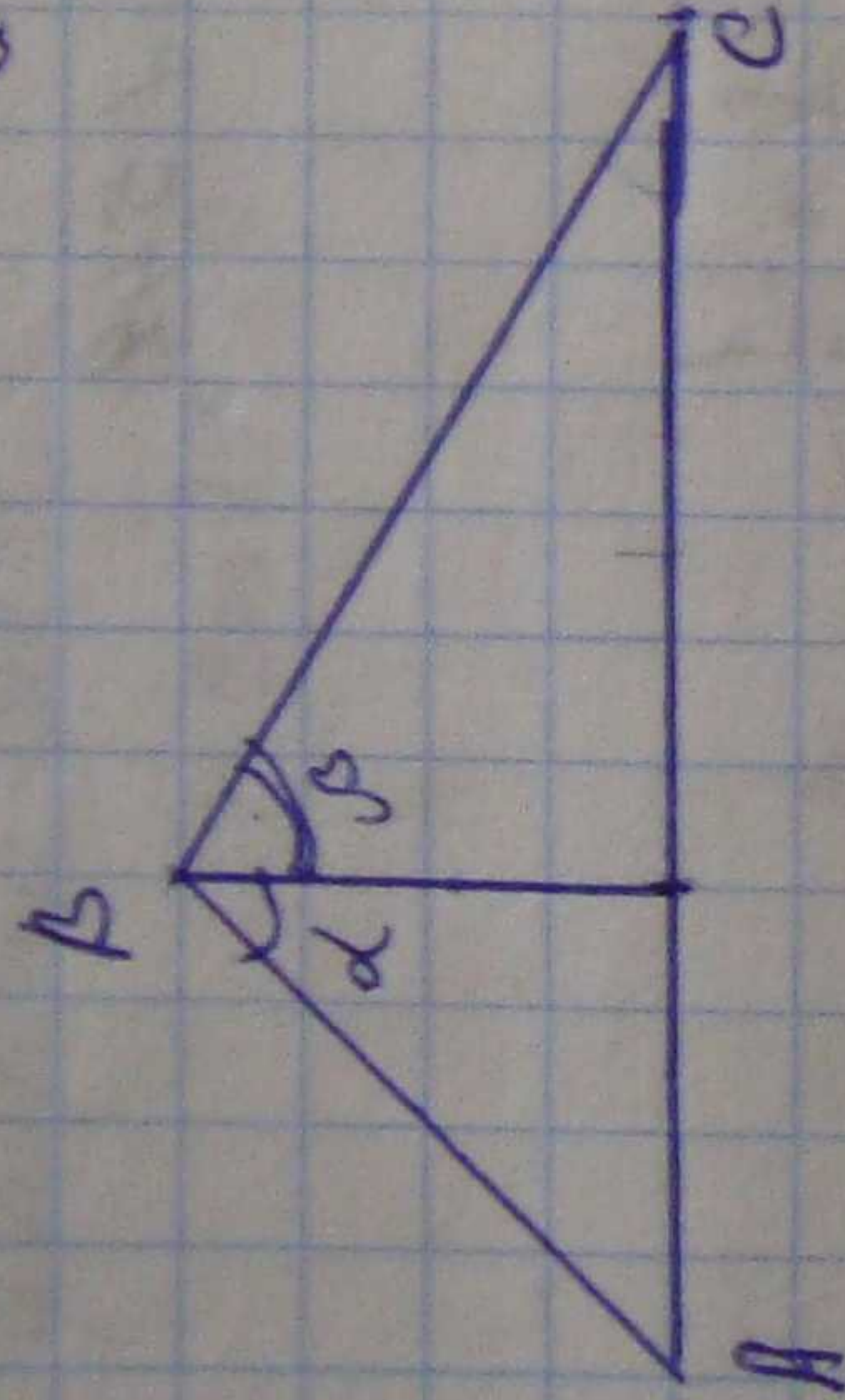
$$Q) \cos 6A = 2\cos^2 3A - 1$$

$$\frac{16}{641}$$

$$\left(\cos \frac{\pi}{7} \cos \frac{4\pi}{7} \cos \frac{5\pi}{7} \right) \approx \frac{1}{8}$$

(2/10/17 554)

2/10/17 543



$$\tan A = \frac{4}{3}$$

$$\tan B = 4$$

2/10/17, 12/12 C = 45°

$$\tan A = \frac{4}{3} \Rightarrow \tan \alpha = \frac{3}{4}$$

$$\tan B = \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = 4$$

$$\frac{\frac{3}{4} + \tan \beta}{1 - \frac{3}{4} \tan \beta} = 4$$

$$\frac{3}{4} + \tan \beta = 4 - \frac{3 \tan \beta}{4}$$

$$\tan \beta + \frac{3 \tan \beta}{4} = 4 - \frac{3}{4}$$

$$\frac{25}{4} \tan \beta = \frac{25}{4}$$

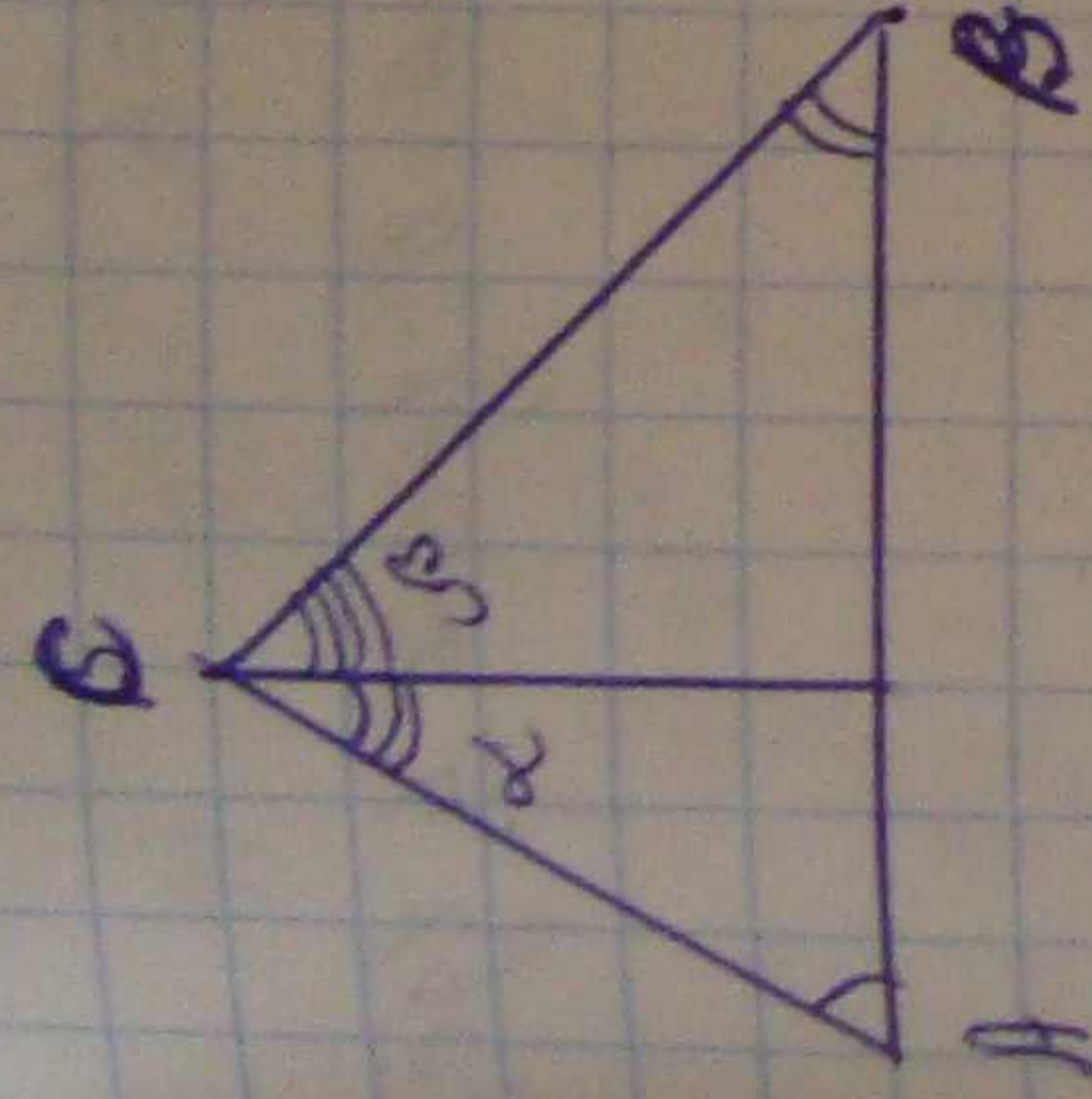
$$\tan \beta = 1$$

$$\tan \beta = \tan C = 1 \Rightarrow \tan C = 1 \Rightarrow C = 45^\circ$$

Задача 544.

A, B, C — углы $< 90^\circ$ в $\triangle ABC$

Доказать, что $\tan A \cdot \tan B > 1$



$$\begin{aligned} \tan A &= \cot B \\ \tan B &= \cot A \end{aligned} \quad \text{или} \quad \tan A \cdot \tan B = \cot A \cdot \cot B$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot A \cot B = \cot(A+B)(\cot A + \cot B) + 1$$

Поскольку $\cot(A+B) > 0$ (так как $A+B < 90^\circ$), то $\cot A \cot B > 1$.

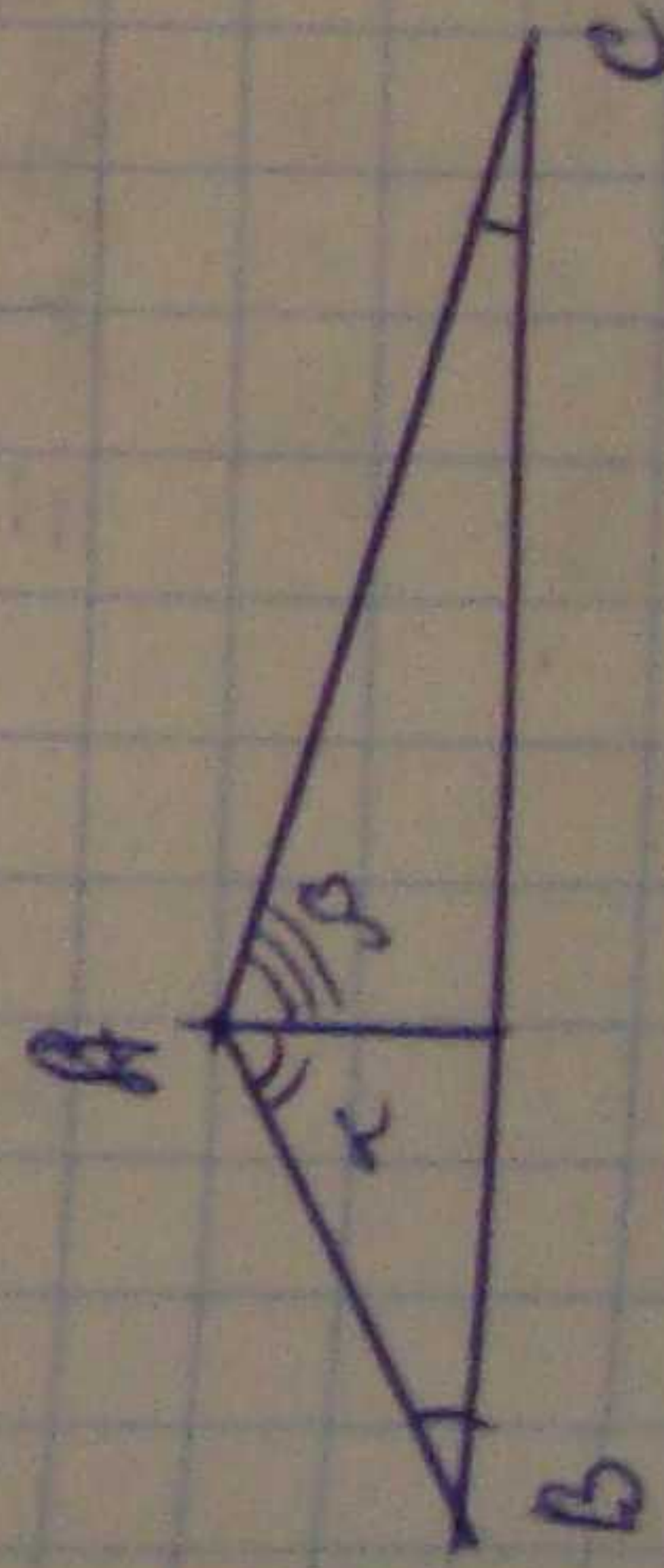
или $\tan A \cdot \tan B > 1$

$$\tan A \cdot \tan B = \cot A \cot B > 1$$

Задача 545

A, B, C — углы $< 90^\circ$ в $\triangle ABC$

$A > 90^\circ$

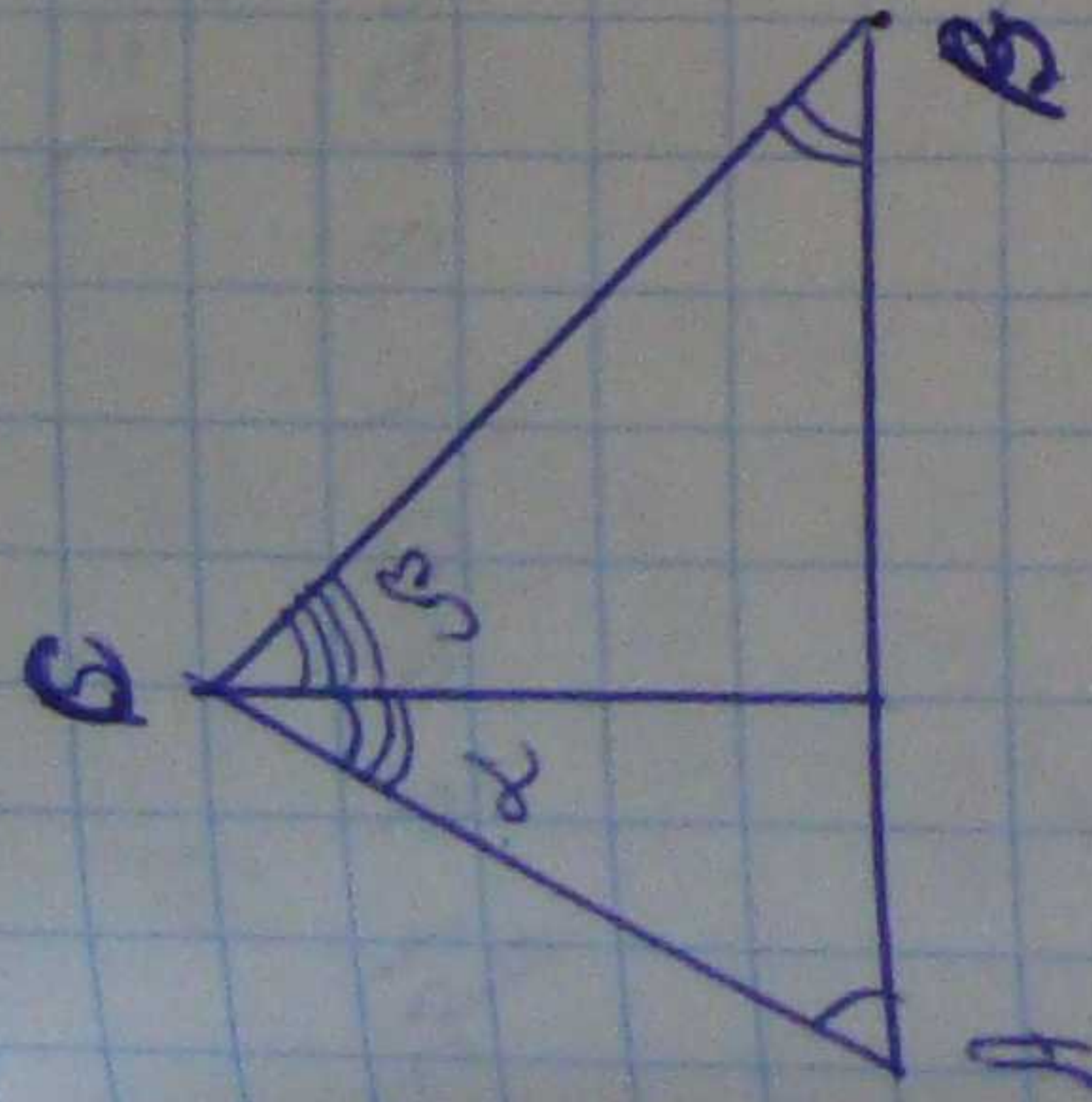


$$\begin{aligned} \tan B \cdot \tan C &= \cot A \cot B = \cot(A+B)(\cot A + \cot B) + 1 = \\ &= \cot A (\cot A + \cot B) + 1 = \cot^2 A + \cot A \cot B + 1 = \end{aligned}$$

2/чурд 544.

$A - 2; B - 2$ и $C - 2 < 90^\circ$ -й

Учир нр. $\frac{\operatorname{tg} A \cdot \operatorname{tg} B}{> 1}$



$$\left. \begin{aligned} \operatorname{tg} A &= \operatorname{ctg} \alpha \\ \operatorname{tg} B &= \operatorname{ctg} \beta \end{aligned} \right\} \text{ нр. } \operatorname{tg} A \cdot \operatorname{tg} B = \operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta$$

$$\operatorname{ctg}(\alpha + \beta) = \frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta}$$

$$\operatorname{ctg} \alpha \operatorname{ctg} \beta = \operatorname{ctg}(\alpha + \beta)(\operatorname{ctg} \alpha + \operatorname{ctg} \beta) + 1$$

Хүчлэх нр $\operatorname{ctg}(\alpha + \beta) > 0$ (нрнхтгал $\operatorname{ctg}(\alpha + \beta) - 2 < 90^\circ$ -й, үүснхдэг гүнзгийрч 5 I дүр).

$\operatorname{ctg} \alpha > 0; \operatorname{ctg} \beta > 0$ нрнх $\operatorname{ctg} \alpha + \operatorname{ctg} \beta > 0$ нр

нр $\operatorname{tg} A \cdot \operatorname{tg} B = \operatorname{ctg} \alpha \operatorname{ctg} \beta > 1$

2/чурд 545

$\angle B - 2$ и $\angle C - 2 < 90^\circ$ -й

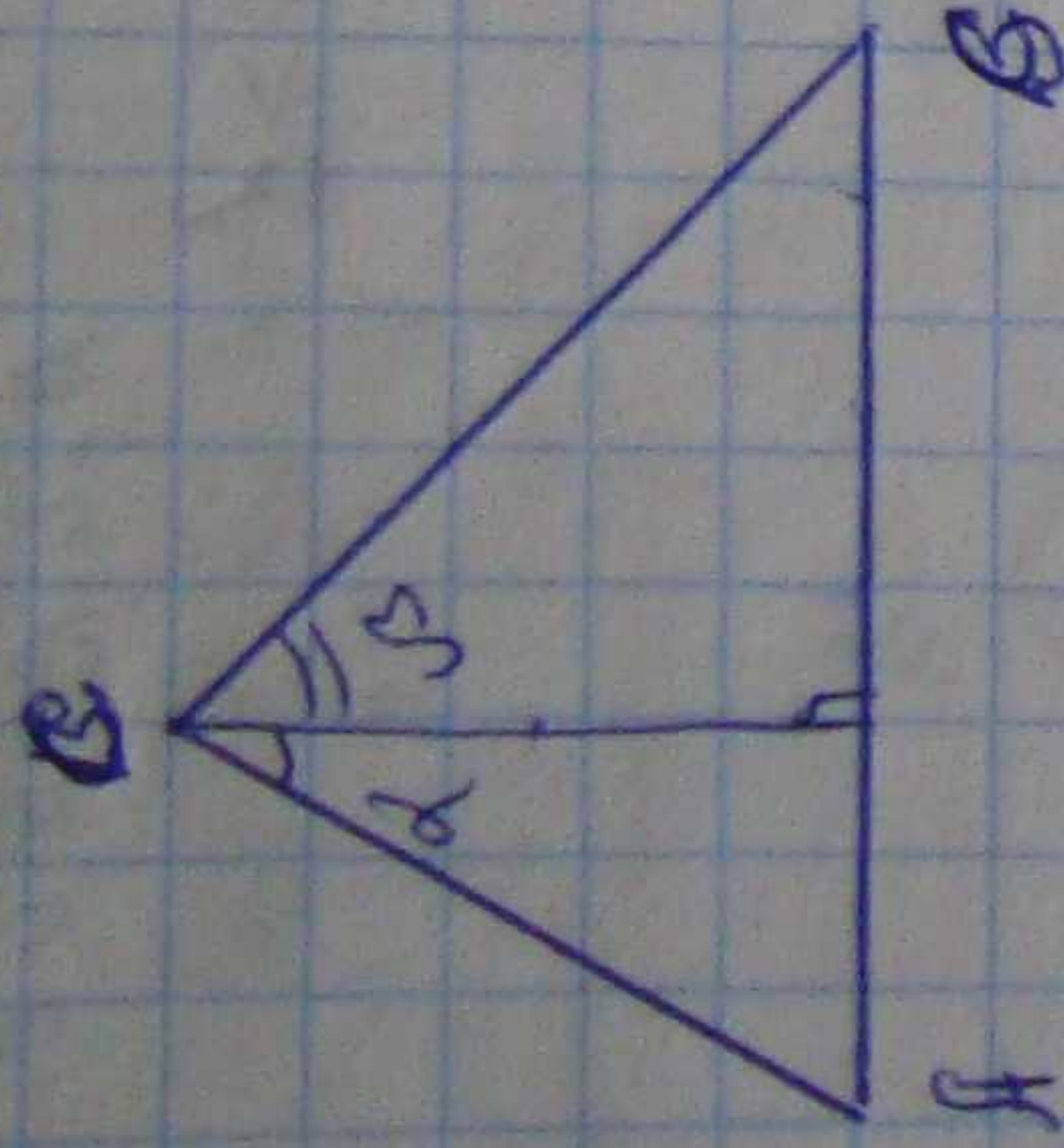
$\angle A > 90^\circ$



Учир нр $\operatorname{tg} B \cdot \operatorname{tg} C < 1$

$$\begin{aligned} \operatorname{tg} B \cdot \operatorname{tg} C &= \operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta = \operatorname{ctg}(\alpha + \beta)(\operatorname{ctg} \alpha + \operatorname{ctg} \beta) + 1 = \\ &= \operatorname{ctg} A(\operatorname{ctg} \alpha + \operatorname{ctg} \beta) + 1 = \operatorname{ctg} A(\operatorname{tg} B + \operatorname{tg} C) + 1 = \end{aligned}$$

2/урд 546



$$\operatorname{tg} A \cdot \operatorname{tg} B = 3$$

$$\operatorname{tg} C \cdot \operatorname{tg} B = 6$$

$$\text{Найти } \operatorname{tg} A, \operatorname{tg} B, \operatorname{tg} C \text{ и } \operatorname{tg} C - 2$$

$$(\operatorname{tg} A \cdot \operatorname{tg} B = \operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta = \operatorname{ctg}(\alpha + \beta))(\operatorname{ctg} \alpha + \operatorname{ctg} \beta) + 1 =$$

$$= \operatorname{ctg} C (\operatorname{ctg} \alpha + \operatorname{ctg} \beta) = 2$$

2/урд 549

$$\sin^3 \alpha = \sin(\alpha + 2\alpha) = \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha =$$

$$= \sin \alpha (\cos^2 \alpha - \sin^2 \alpha) + \cos \alpha (2 \sin \alpha \cos \alpha) =$$

$$= \sin \alpha (1 - 2 \sin^2 \alpha) + 2 \sin \alpha (1 - \sin^2 \alpha) = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos^3 \alpha = \cos(\alpha + 2\alpha) = \cos \alpha \cos 2\alpha - \sin \alpha \sin 2\alpha =$$

$$= \cos \alpha (\cos^2 \alpha - \sin^2 \alpha) - 2 \sin \alpha \cos \alpha =$$

$$= \cos \alpha (2 \cos^2 \alpha - 1) - 2 \cos \alpha (\cos^2 \alpha - 1) =$$

$$= 2 \cos^3 \alpha - \cos \alpha - 2 \cos^3 \alpha + 2 \cos \alpha =$$

$$= \cos \alpha$$

4)

$$z =$$

$$\operatorname{tg} 45^\circ$$

$$\operatorname{tg} 1$$

$$508$$

$$\sin 1 - 3$$

$$z = 2 \sin$$

$$z =$$

$$\sin 28^\circ$$

$$\sin 28^\circ$$

Анализ

Задача 536

$$4) \quad \lg 1^\circ \cdot \lg 3^\circ \cdot \lg 5^\circ \dots \lg 89^\circ =$$

$$= (\lg 1^\circ \cdot \lg 89^\circ) (\lg 3^\circ \cdot \lg 87^\circ) \cdot (\lg 5^\circ \cdot \lg 85^\circ) \dots (\lg 43^\circ \cdot \lg 47^\circ) \cdot \lg 45^\circ = (\lg 1^\circ \lg 1^\circ) (\lg 3^\circ \cdot \lg 3^\circ) (\lg 5^\circ \cdot \lg 5^\circ) \dots (\lg 43^\circ \cdot \lg 43^\circ)$$

$$\lg 1 = 1$$

Задача

538.

$$\sin 1 - \sin 2 = \left(\sin \frac{1}{3,14} - \sin \frac{2}{3,14} \right) \approx \sin$$

$$= 2 \sin \frac{1-2}{2} \cos \frac{3}{2} = -2 \sin \frac{1}{2} \cos \frac{3}{2}$$

$$> 0 > 0$$

$$= -2 \sin 0,5 \cos 1,5 < 1$$

$$\sin > 0$$

Задача 542

$$\sin 25^\circ + \cos 15^\circ > 1$$

$$\sin 25^\circ + \sin 75^\circ = 2 \sin 50^\circ \cos 25^\circ$$

$$50^\circ > 45^\circ < 50^\circ$$

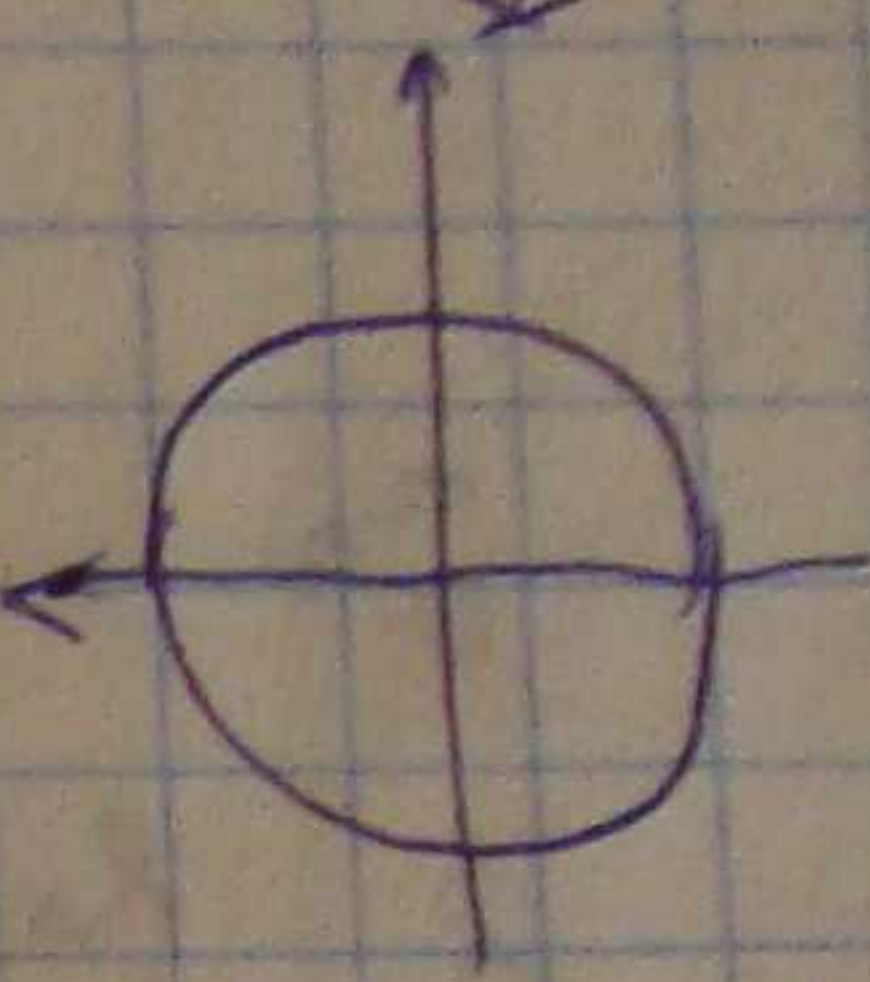
Задача

$$\sin 50^\circ > \sin 45^\circ \quad \sin 50^\circ > \frac{\sqrt{2}}{2}$$

$$25 < 45^\circ$$

$$\cos 25^\circ > \cos 45^\circ$$

$$\cos 25^\circ > \frac{\sqrt{2}}{2}$$



$$\cos 25^\circ \sin 50^\circ = \frac{1}{2}$$

$$2 \cos 25^\circ \sin 50^\circ = 1$$

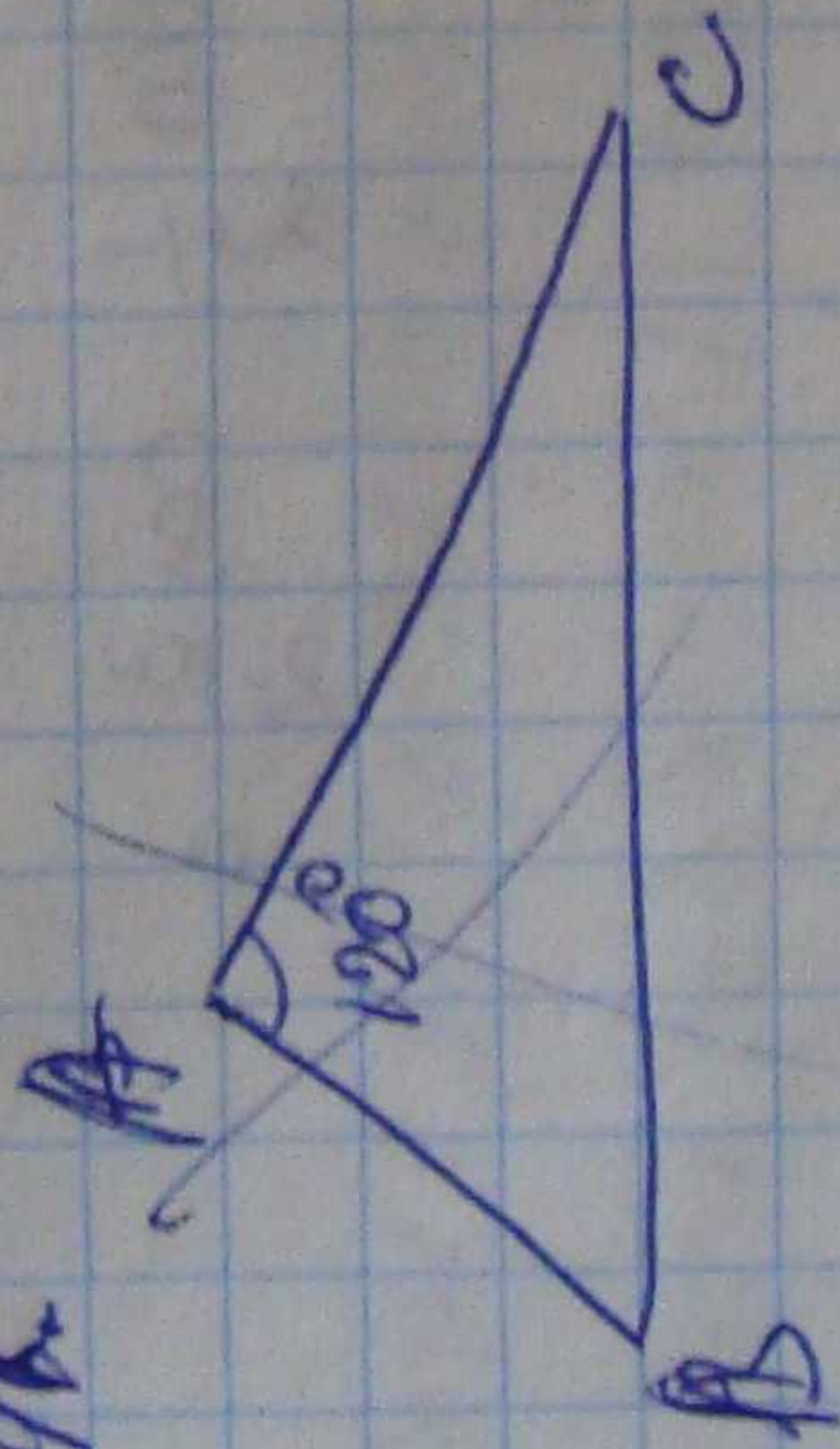
Quind. 543

the

$$\operatorname{tg} C = \operatorname{tg} (180^\circ - (A+B)) = -\operatorname{tg} (A+B)$$

$$-\operatorname{tg} (A+B) = \frac{\operatorname{tg} A + \operatorname{tg} B}{1 - \operatorname{tg} A \operatorname{tg} B} = -\frac{\frac{4}{3} + \frac{7}{3}}{1 - \frac{28}{9}} = \frac{11}{1-28} = \frac{11}{-27}$$

543



$$\angle A = 120^\circ$$

$$\cos B + \cos C = \sqrt{3}$$

$$\angle B = ? \quad \angle C = ?$$

$$(\cos A (180^\circ - 60^\circ))$$

$$B+C = 180^\circ - 120^\circ = 60^\circ$$

$$2 \cos \frac{B+C}{2} \cos \frac{B-C}{2} = 2 \cos 30^\circ \cos \frac{B-C}{2} = \sqrt{3}$$

$$\sqrt{3} \cos \frac{B-C}{2} = 1$$

$$B-C = 0$$

$$B=C = 30^\circ$$

550/ $\sin \alpha = \frac{3}{5}$ $\frac{\pi}{2} < \alpha < \pi$

$$\sin 3\alpha$$

$$\sin 4\alpha$$

$$\sin \frac{\alpha}{2}$$

$$\cos 3\alpha$$

$$\sin 3\alpha = \sin(\alpha + 2\alpha) = \sin \alpha \cos 2\alpha + \cos \alpha \sin 2\alpha =$$

$$= \sin \alpha (\sin \alpha \cos^2 \alpha - \sin^3 \alpha) + 2 \cos^2 \alpha \sin \alpha =$$

$$= \sin \alpha ($$

$$\sin 4\alpha = 2 \sin 2\alpha \cos 2\alpha = 2 (\sin \alpha \cos \alpha (\cos^2 \alpha - \sin^2 \alpha)) =$$

$$= 4 \sin \alpha \cos \alpha (1 - 2 \sin^2 \alpha)$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$4 \cdot \frac{3}{5} \cdot \left(-\frac{4}{5} \cdot \left(1 - \frac{18}{25} \right) \right) = 12 \cdot \frac{7}{25} = \frac{84}{25}$$

$$= \frac{7}{25} \cdot \frac{12}{5}$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} =$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos 3\alpha$$

552.

nhay.

$$p) \alpha, \beta \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$$

$$\text{thay, np } \frac{\cos \alpha + \beta}{2}, \frac{\cos \alpha + \cos \beta}{2}$$

$$\frac{\cos \alpha + \cos \beta}{2}, \frac{2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}}{2}$$

$$2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$$

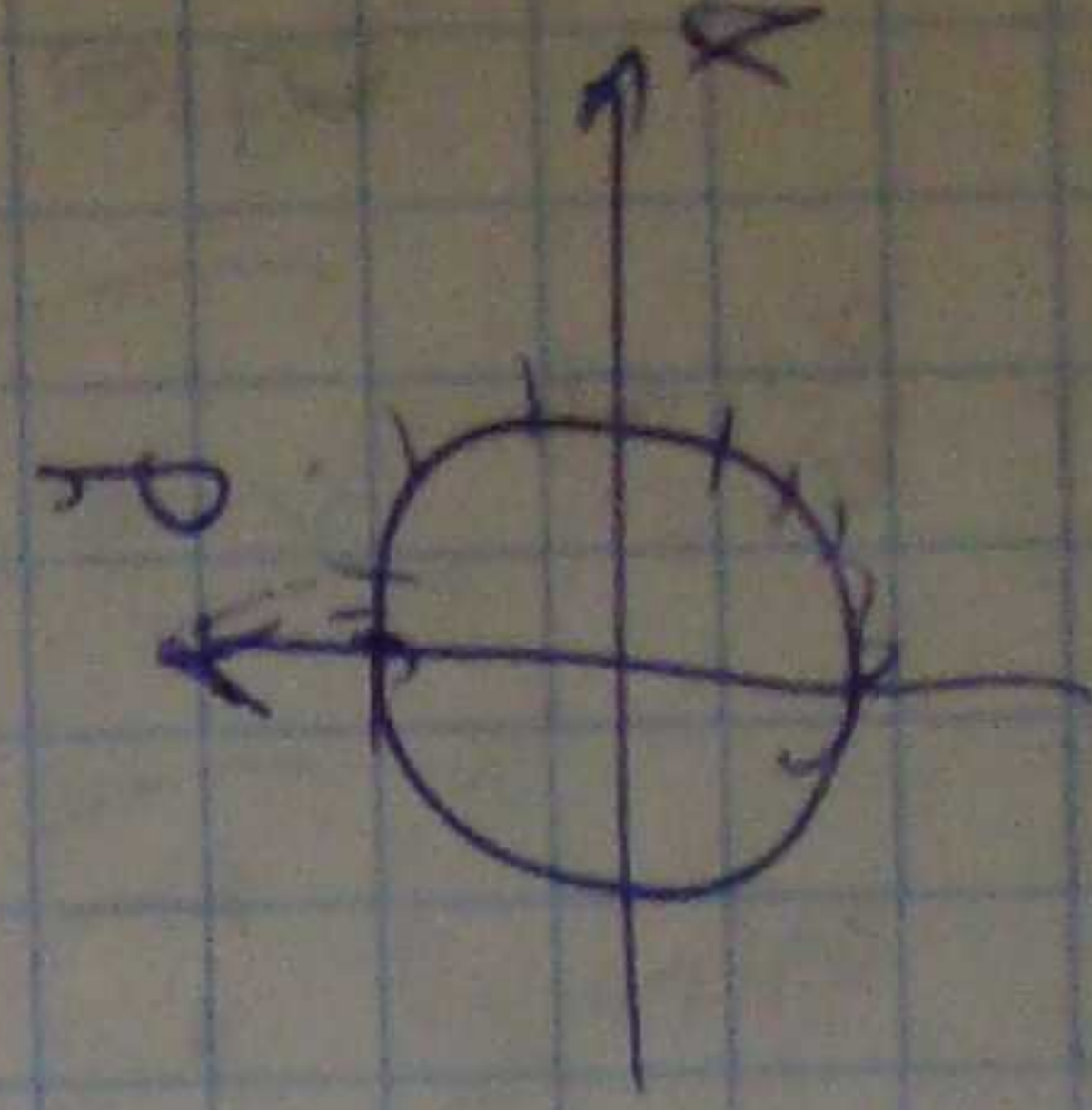
$$-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2} \leq -\beta \leq \frac{\pi}{2}$$

$$-\pi \leq \alpha - \beta \leq \pi$$

$$-\frac{\pi}{2} \leq \frac{\alpha - \beta}{2} \leq \frac{\pi}{2}$$



$$d) \frac{2 \cos \alpha \sin \beta + \sin(\alpha - \beta)}{\cos(\alpha - \beta) - 2 \cos \alpha \cos \beta} z$$

$$z \cdot \frac{2 \cos \alpha \sin \beta + \cos \alpha \sin \beta \sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta - 2 \cos \alpha \cos \beta}$$

$$z \cdot \frac{\cos \alpha \sin \beta + \sin \alpha \cos \beta}{\sin \alpha \sin \beta - \cos \alpha \cos \beta}$$

$$z \cdot \frac{\sin(\alpha + \beta)}{-\cos(\alpha + \beta)} z = -\operatorname{tg}(\alpha + \beta)$$

$$7) \operatorname{tg} x \cdot \operatorname{tg}(x + 60^\circ) - \operatorname{tg}(x - 60^\circ) = -\operatorname{tg} 3x$$

$$(\operatorname{tg} x \operatorname{tg} x + \operatorname{tg} 60^\circ)$$

$$+ \operatorname{tg} x \cdot \frac{\sin(x + 60^\circ)}{\cos(x + 60^\circ)} \cdot \frac{\sin(x - 60^\circ)}{\cos(x - 60^\circ)} z$$

$$z \cdot \operatorname{tg} x \cdot \frac{\sin(x + 60^\circ) \sin(x - 60^\circ)}{\cos(x + 60^\circ) \cos(x - 60^\circ)} z$$

$$z \cdot \operatorname{tg} x \cdot \frac{\cos 120^\circ - \cos 2x}{\cos 120^\circ + \cos 2x} z$$

$$z \cdot \operatorname{tg} x \cdot \frac{-\frac{1}{2} - \cos 2x}{-\frac{1}{2} + \cos 2x} z = \frac{1 + 2 \cos 2x}{1 + 2 \cos 2x - 1} \cdot \frac{\sin x}{\cos x} z$$

$$z = \frac{\sin x + 2 \sin x \cos 2x}{2 \cos x \cos 2x - \cos x} z = \frac{\sin x + \sin(x - 2x) + \sin 3x}{\cos(x - 2x) + \cos 3x - \cos x} z$$

$$\frac{\sin x - \sin x + \sin 3x}{\cos x + \cos 3x - \cos x} = \frac{\sin 3x}{\cos 3x}$$

Wichtig: $\sin 3x = 3 \sin x - 4 \sin^3 x$

$$y = \sin x$$

$$\sin(\sin x) = \sin x$$

$$2. \ E(\sin x) = [-1, 1]$$

$$3. \ \sin(-x) = -\sin x$$

$$\forall x \in \mathbb{R} \ (\sin x)$$

es gilt $\sin x \in [-1, 1]$ für alle $x \in \mathbb{R}$

$$4. \ \sin(x + 2\pi k) = \sin x \quad k \in \mathbb{Z}$$

$$\sin(x + 2\pi) = \sin x \quad \sin(-x) = -\sin x$$

Periodizität

$$[-\pi, \pi]$$

5. $\sin x > 0 \iff x \in (2\pi n, 2\pi(n+1)) \quad n \in \mathbb{Z}$

$$\sin x < 0 \iff x \in (2\pi(n+1), 2\pi(n+2)) \quad n \in \mathbb{Z}$$

$$n \in \mathbb{Z}$$

$$\sin x = 0 \iff x \in \pi \mathbb{Z}$$

$$6. \ \sin x = 0$$

$$x \in \pi \mathbb{Z} \quad k \in \mathbb{Z}$$

7. $\sin x \in [-1, 1]$ (aus $\sin^2 x + \cos^2 x = 1$)

$$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$x \in \left[\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi + \pi \right]$$

$$x \in \left[\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi \right] \downarrow$$

$$n \in \mathbb{Z}$$

$$8. \sin x = 1 \quad x_{\max} = \frac{\pi}{2} + 2n\pi$$

$$y_{\max} = 1$$

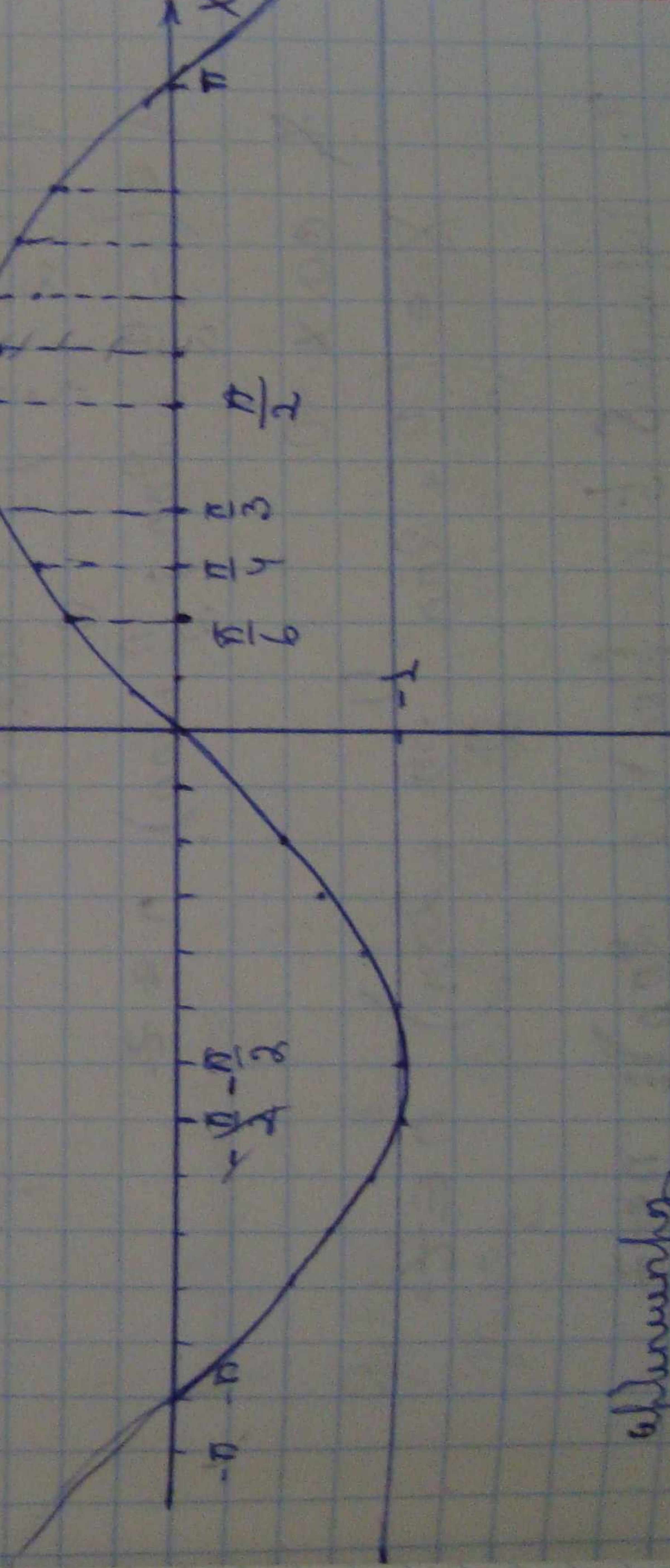
$$\sin x = -1 \quad x_{\min} = \frac{3\pi}{2} + 2n\pi$$

$$n \in \mathbb{Z}$$

$$y_{\min} = -1$$

y

1



aftermath

$$x \in \left[\frac{\pi}{2} + 2\pi n, \frac{\pi}{2} + 2\pi n \right] \uparrow$$

$$n \in \mathbb{Z}$$

$$x \in \left[\frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n \right] \downarrow$$

$$n \in \mathbb{Z}$$

$$8. \sin x = 1 \quad x_{\max} = \frac{\pi}{2} + 2\pi n$$

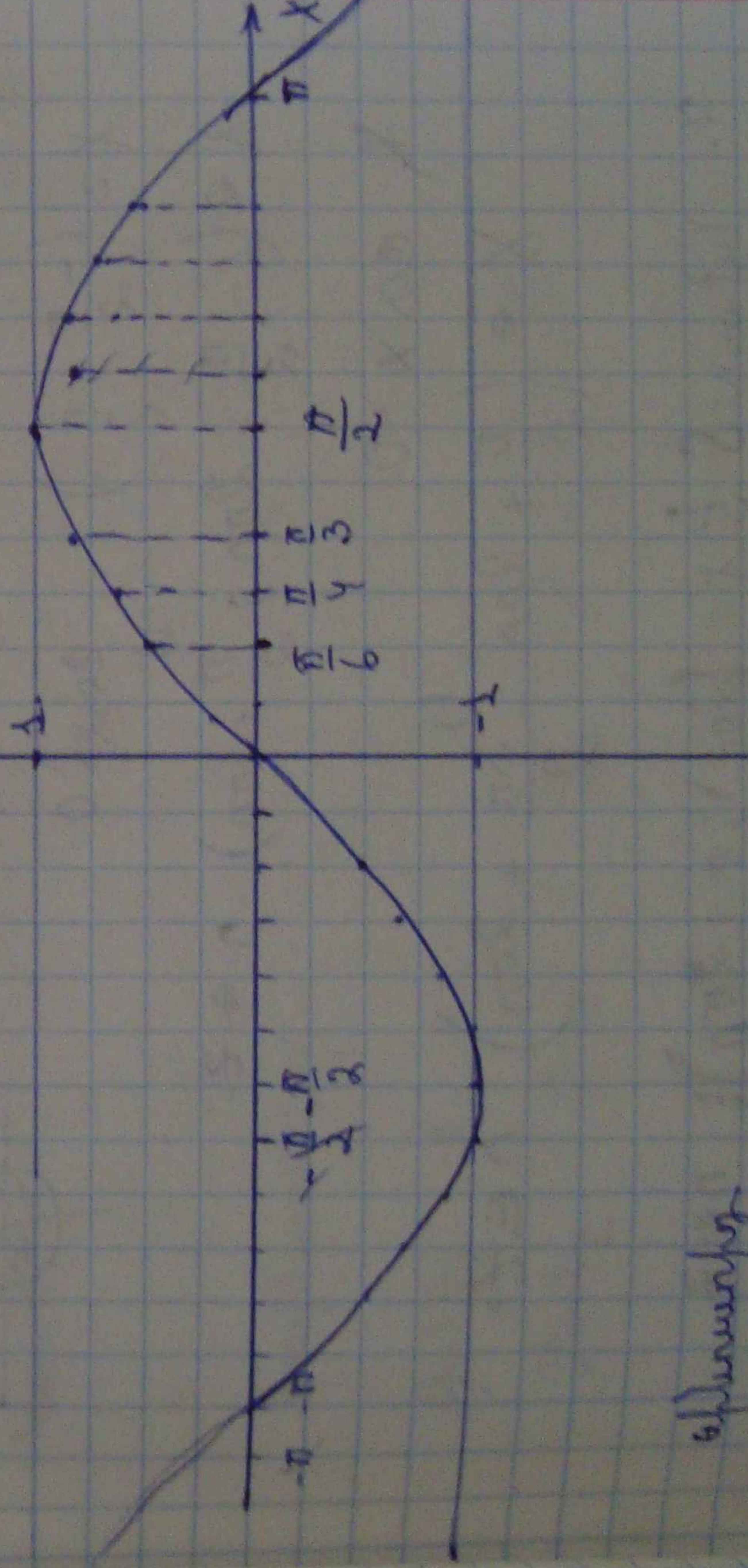
$$y_{\max} = 1$$

$$\sin x = -1 \quad x_{\min} = -\frac{\pi}{2} + 2\pi n$$

$$n \in \mathbb{Z}$$

$$y_{\min} = -1$$

xy



sinus

$$y = \cos x$$

$$1. \cos(\cos x) = \mathbb{R}$$

$$2. \mathbb{R}(\cos x) = [-1, 1]$$

$$3. \cos(-x) = \cos x \quad \text{yay! } 5.$$

$$4. \cos(x + 2\pi n) = \cos x \quad n \in \mathbb{Z}$$

$$5. T = 2\pi n$$

hubschwerk y-geradenperioden 2π

5. f. univ.-periode

$$\cos x = 0$$

$$x = \frac{\pi}{2} + \pi k \quad k \in \mathbb{Z}$$

6. 1-2. hump. Spiegelungsfkt

$$x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \quad \cos x > 0$$

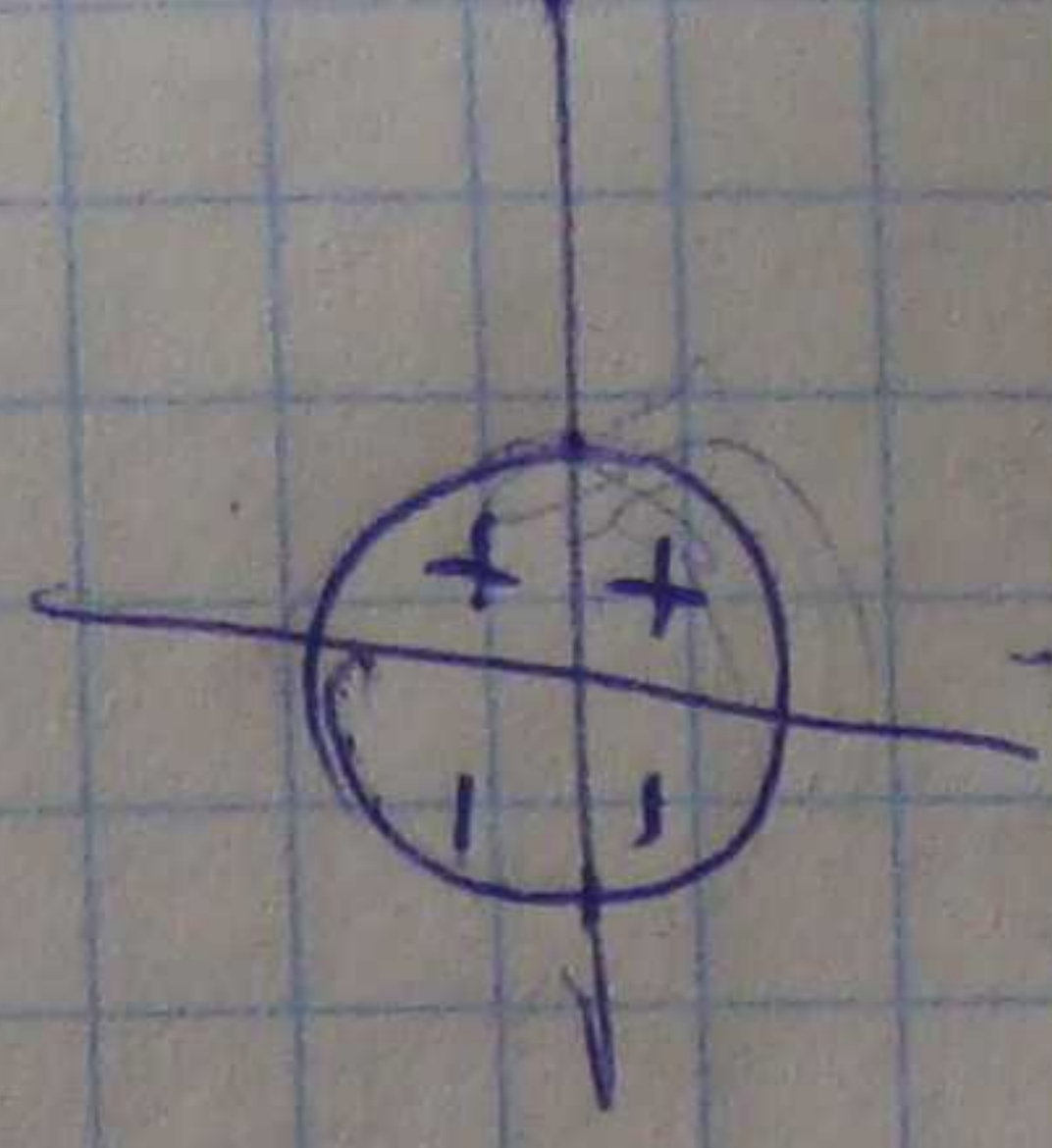
$$x \in \left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right) \quad n \in \mathbb{Z}$$

$$\cos x < 0$$

$$x \in \left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right) \quad n \in \mathbb{Z}$$

7. Infimums 5 $x \in [2\pi n; \pi + 2\pi n) \quad n \in \mathbb{Z}$

Unions $x \in [\pi + 2\pi n; 2\pi n) \uparrow$



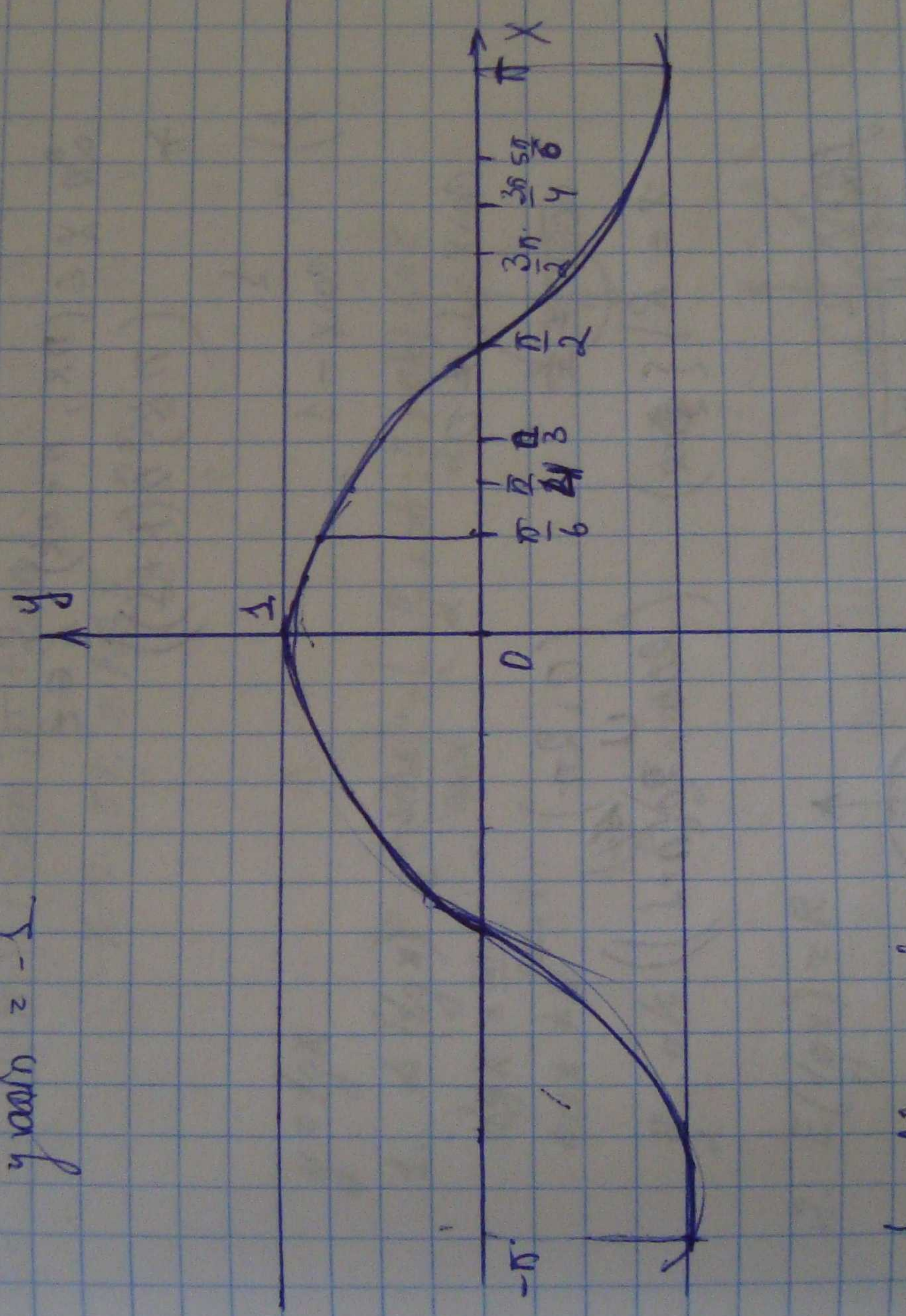
$$8. \cos x = 1$$

$$x_{\max} = 2\pi n \quad n \in \mathbb{Z}$$

$$y_{\max} = 1$$

$$x_{\min} = \pi + 2\pi n \quad n \in \mathbb{Z}$$

$$y_{\min} = -1$$



Graphieren.

Graph, \$

Wurde Graph in 8.5

on 279-285.

fund.

$$f(x) = \frac{1}{\sin x}$$

$$\sin x = 0, x \in \mathbb{R}$$

$$x \neq \pi k$$

$$2\pi x \in (\pi k; \pi + \pi k) \quad k \in \mathbb{Z}$$

$$\pi \quad (\pi k; \pi(k+1))$$

$$y = \frac{1}{\cos x - 1}$$

$$\cos x - 1 \neq 0$$

$$\cos x \neq 1$$

$$x \in \mathbb{R} \setminus \{2\pi k\}$$

$$(0; 2\pi) \cup (2\pi n; 2\pi(n+1))$$

$$f(x) = \frac{2}{\sqrt{-\sin x}}$$

$$-\sin x > 0$$

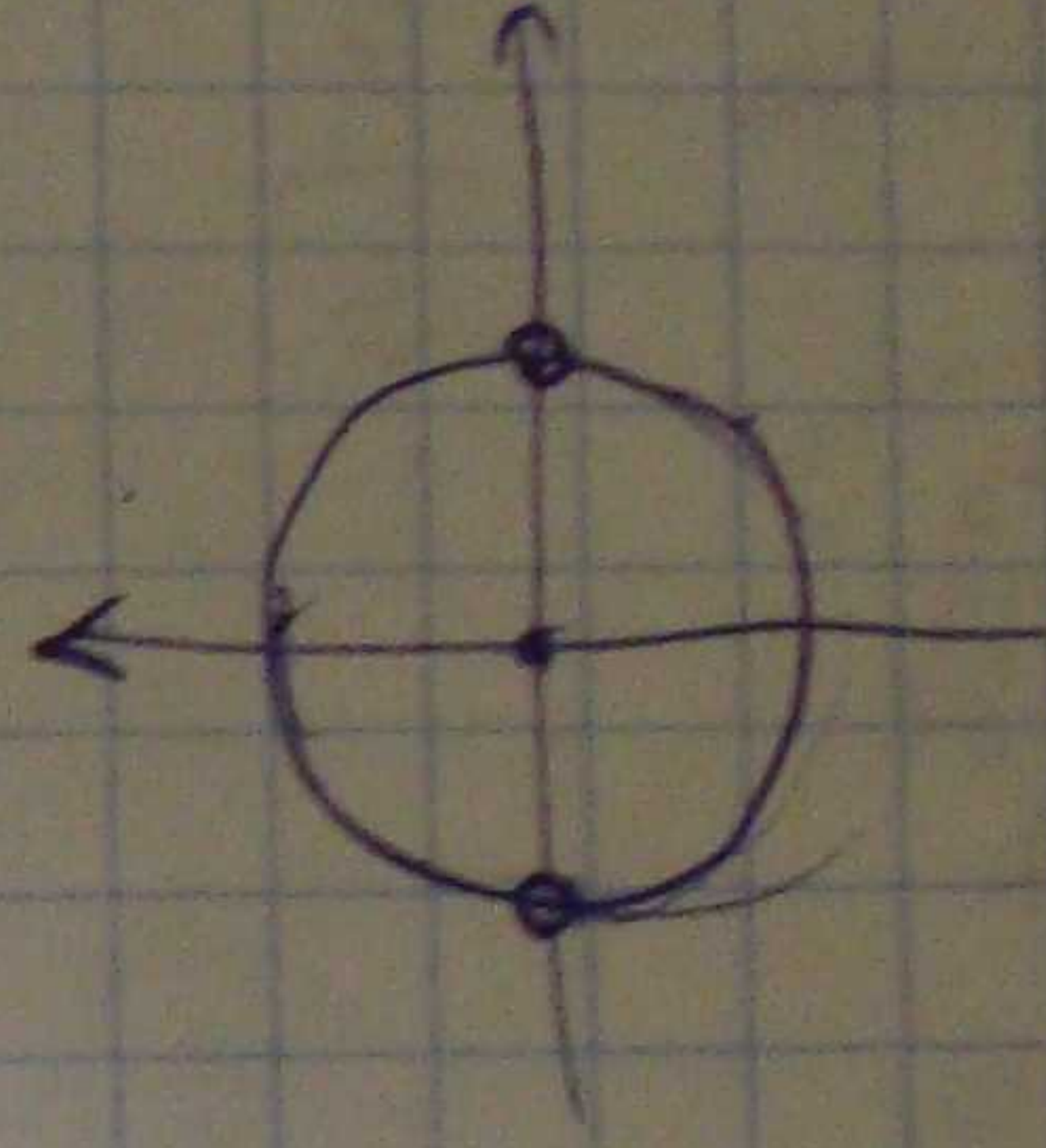
$$\sin x < 0$$

$$(\pi; 2\pi)$$

$$(\pi + 2\pi n; 2\pi + 2\pi n)$$

$$(\pi(1+2n); 2\pi(1+n))$$

$$(\pi + 2\pi n; 2\pi(n+1))$$



$$\operatorname{tg} f(x) = \sqrt{\sin x \cdot \cos x}$$

$$\sin x \cos x \geq 0$$

$$\frac{1}{2} \sin 2x \geq 0$$

$$\sin 2x \geq 0$$

$$(0, \pi) \cup (2\pi, \pi + 2\pi)$$

$$x \in \pi n \leq x \leq \frac{\pi}{2} + \pi n$$

$$y = \operatorname{tg} x$$

$$1. \infty (\operatorname{tg} x) = \frac{\sin x}{\cos x} = \left(\frac{\pi}{2} + \pi k; \frac{\pi}{2} + \pi k \right) k \in \mathbb{Z}$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$\cos x \neq 0$$

$$\frac{\pi}{2} + \pi k \quad R / \frac{\pi}{2} + \pi k$$

$$2. E(\operatorname{tg} x) = \mathbb{R}$$

$$3. \operatorname{tg}(-x) = -\operatorname{tg} x$$

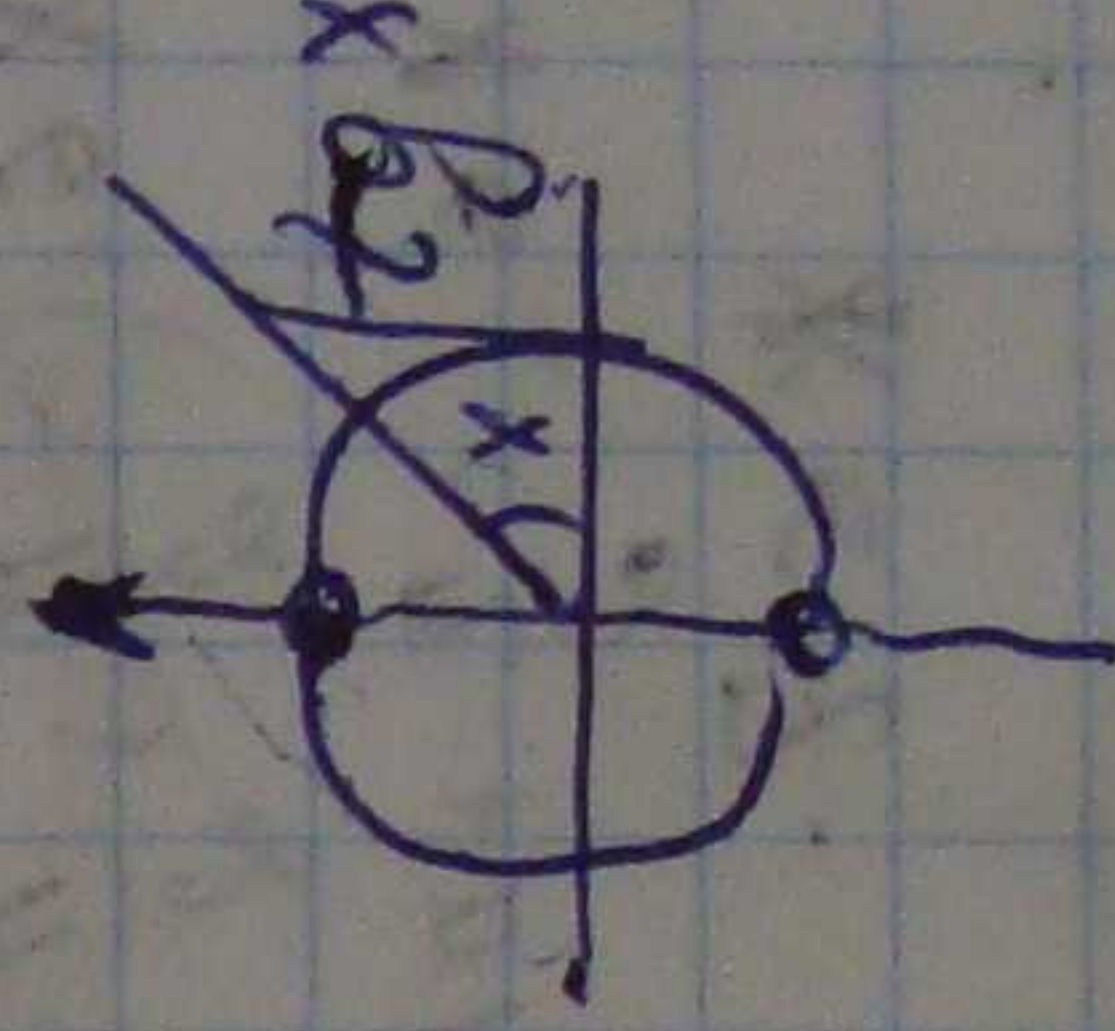
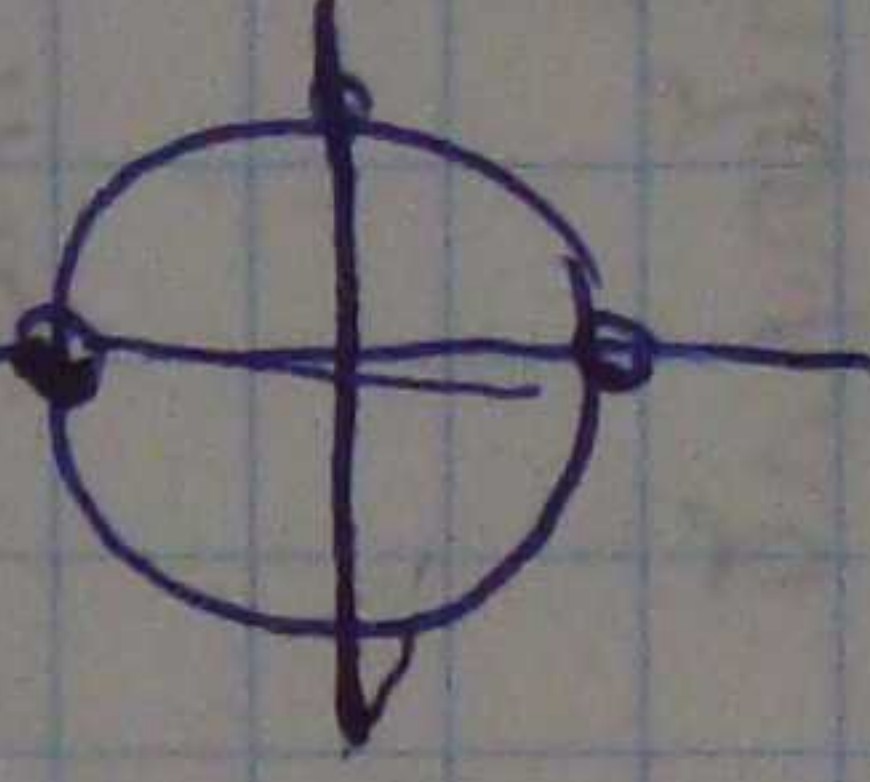
$$\operatorname{tg}(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\operatorname{tg} x; x \in \mathbb{R}$$

$$\operatorname{tg} x - h \text{ нмдвр. } \{ 0 \} \text{ змвр.}$$

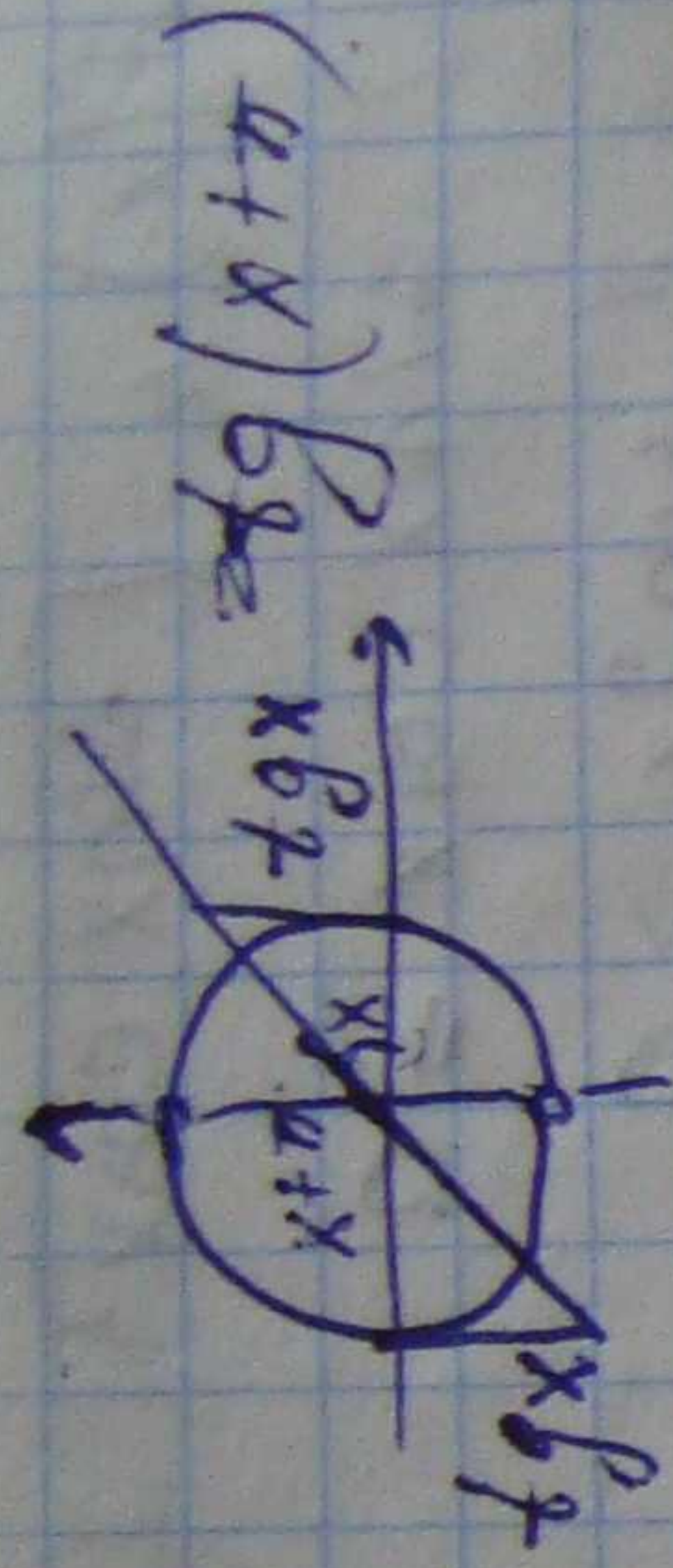
$$4. \operatorname{tg}(x + \pi k) = \operatorname{tg} x$$

$$T = \pi k; k \neq 0, k \in \mathbb{Z}$$

$$T_{\operatorname{tg}} = \pi$$



$$\operatorname{tg}(x+\pi)$$



$$5. \operatorname{tg} x > 0.$$

$x \in (0; \frac{\pi}{2})$ при этом $\operatorname{tg} x > 0$ $x \in (\pi k; \frac{\pi}{2} + \pi k)$

$$\operatorname{tg} x < 0$$

$x \in (-\frac{\pi}{2} + \pi k; \pi k)$

$$6. \operatorname{tg} x = 0.$$

$$x = \pi k$$



7. Интервалы. Случай $\operatorname{tg} x > 0$

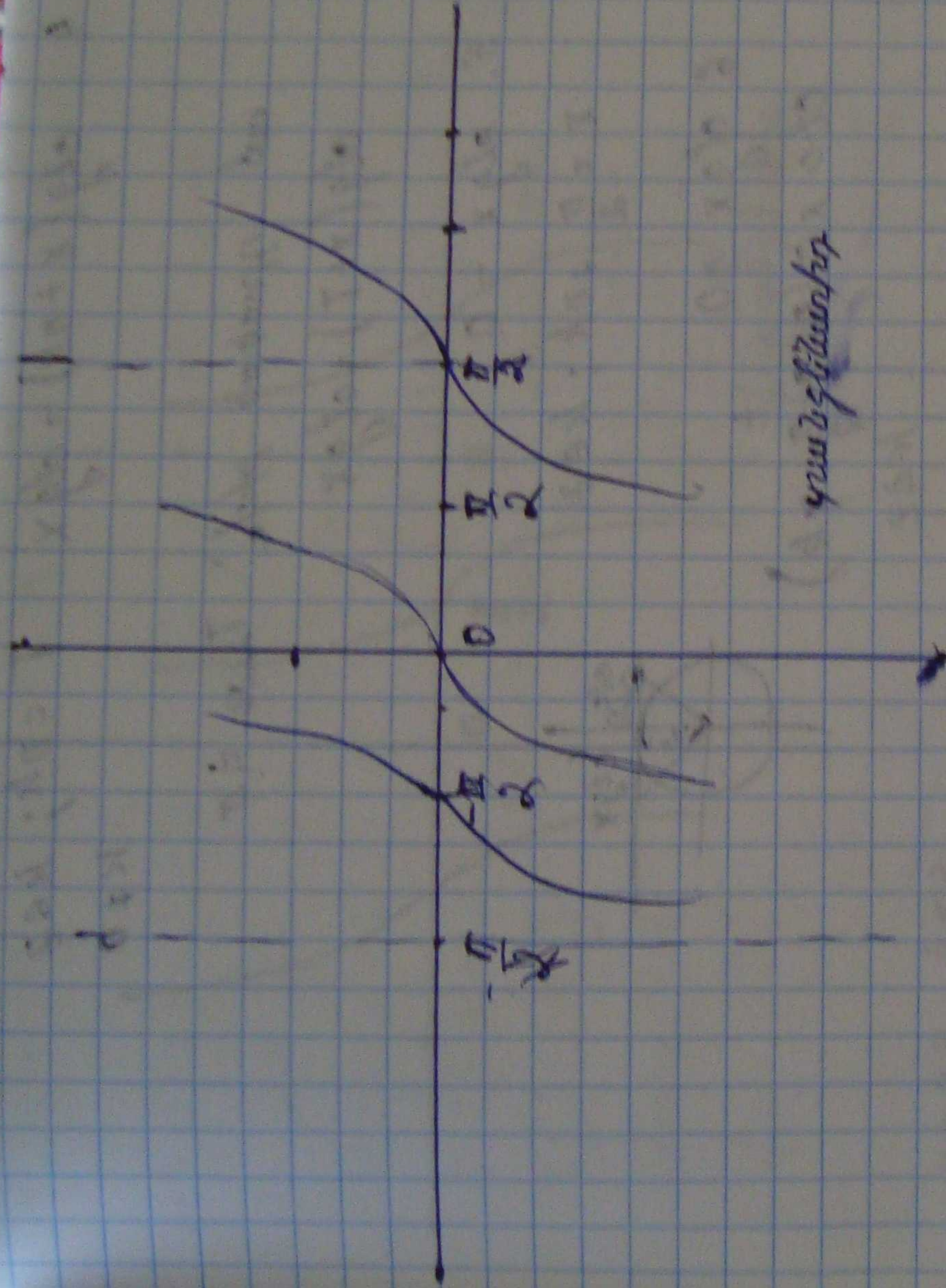
$$x \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$x \in (-\frac{\pi}{2} + \pi k; \frac{\pi}{2} + \pi k)$$

$$x \in \mathbb{R}$$

$$\left. \begin{array}{l} x \rightarrow -\frac{\pi}{2} \\ \operatorname{tg} x \rightarrow +\infty \end{array} \right\}$$

$$\left. \begin{array}{l} x \rightarrow -\frac{\pi}{2} \\ \operatorname{tg} x \rightarrow -\infty \end{array} \right\}$$



համաձայն

$$x = \frac{\pi}{2} + \pi k$$

Հ. Կ համաձայն հոգեւոր ուղղութիւն արեւմտական

$$y = ctg x$$

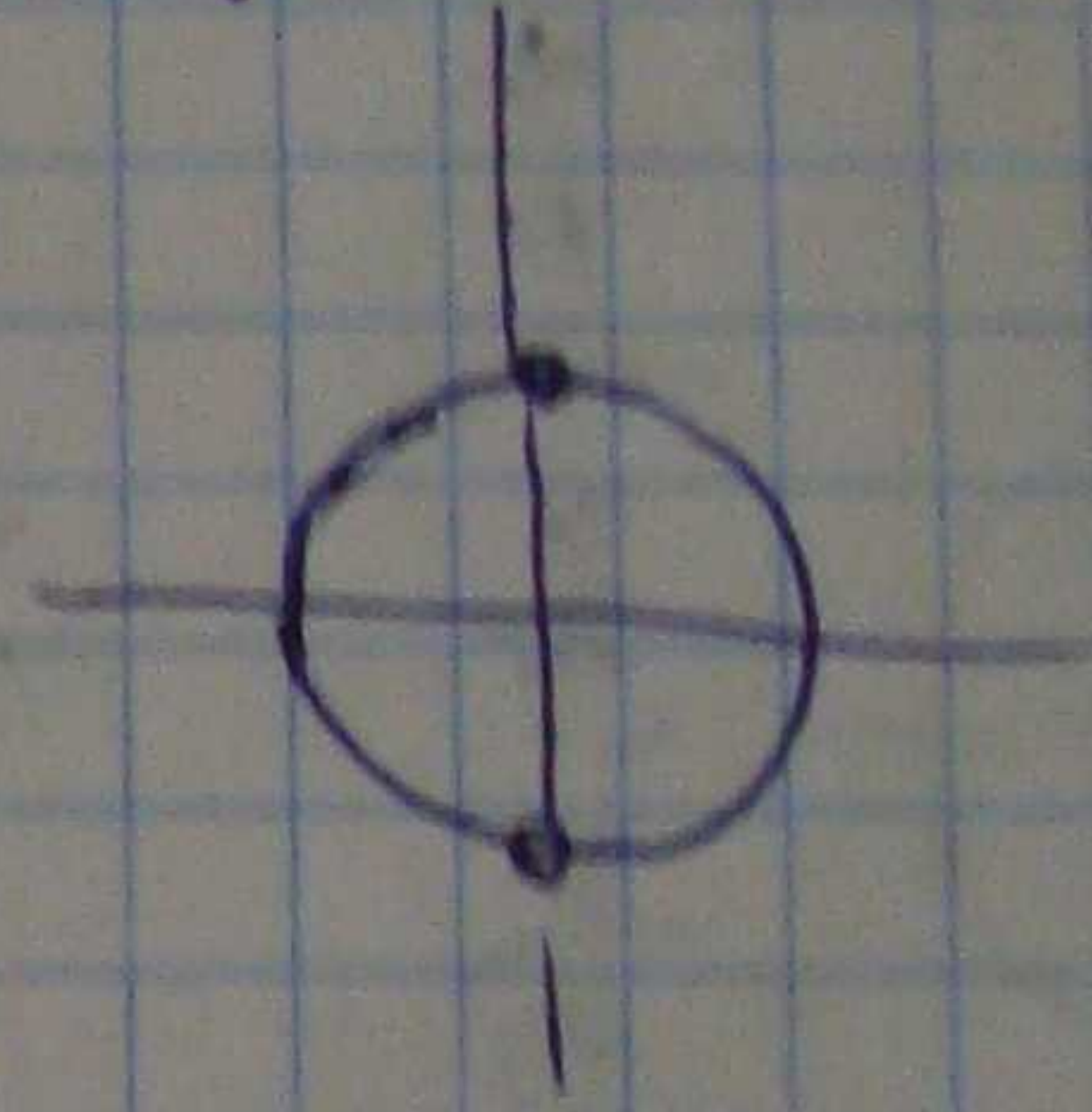
$$1. \in (ctg x) \in \{ \pi k, k \in \mathbb{Z} \}$$

$$= (\pi k; \pi(k+1)), k \in \mathbb{Z}$$

$$2. \mathbb{R} \setminus (ctg x) = \mathbb{R}$$

$$3. ctg(-x) = -ctg x$$

համաձայն յանգ



$$4. \operatorname{ctg}(x + \pi k) = \operatorname{ctg} x \quad T = \pi k; k \in \mathbb{Z} \\ k \neq 0$$

ув. периодическое: $T_{\operatorname{ctg}} = \pi$

$$\operatorname{ctg}(x + T) = \operatorname{ctg} x$$

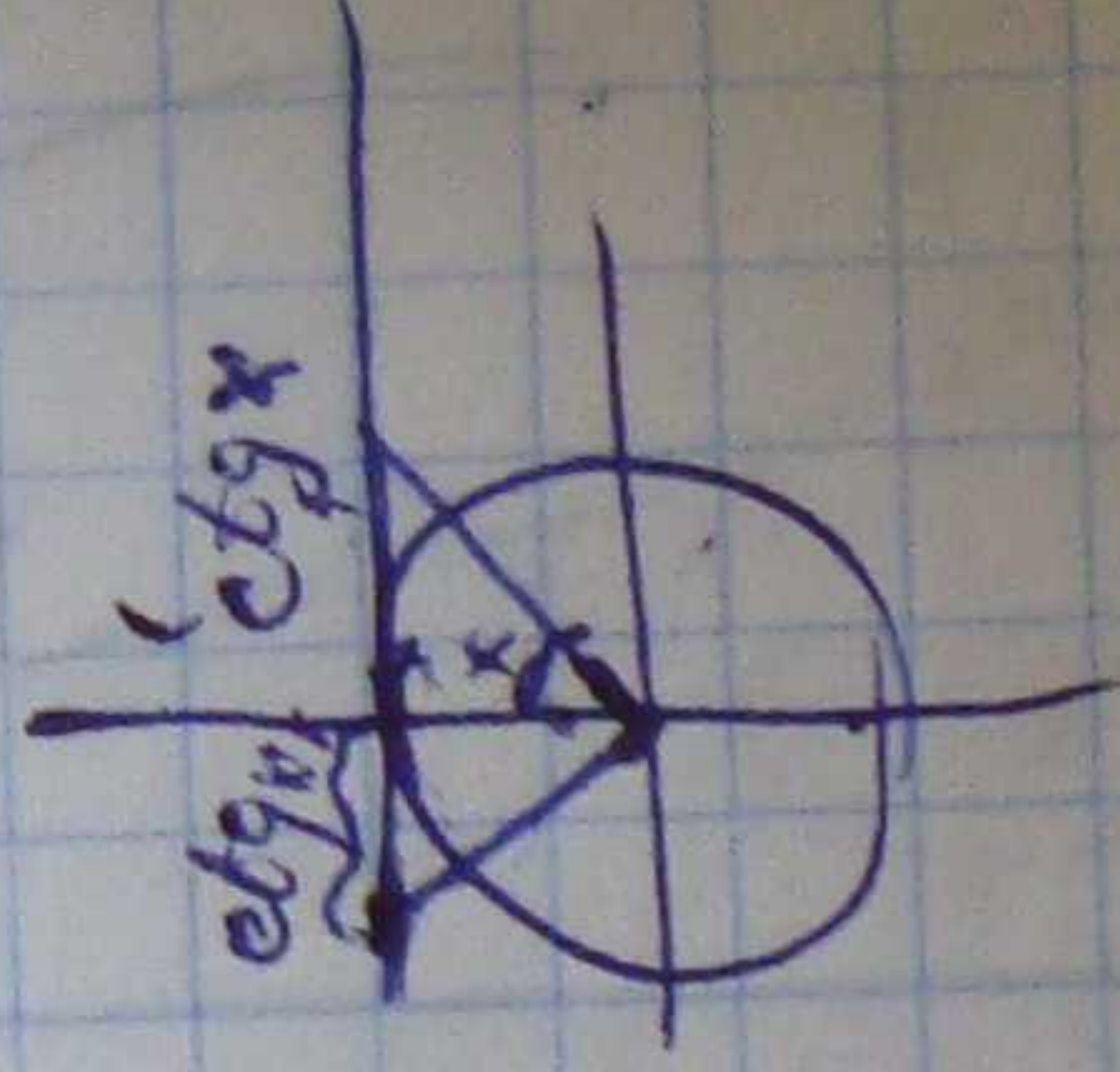
$$5. \operatorname{ctg} x = 0$$

$$x = \frac{\pi}{2} + \pi k; k \in \mathbb{Z}$$

$$6. \operatorname{ctg} x > 0$$

$$x \in \left(\pi k, \frac{\pi}{2} + \pi k \right)$$

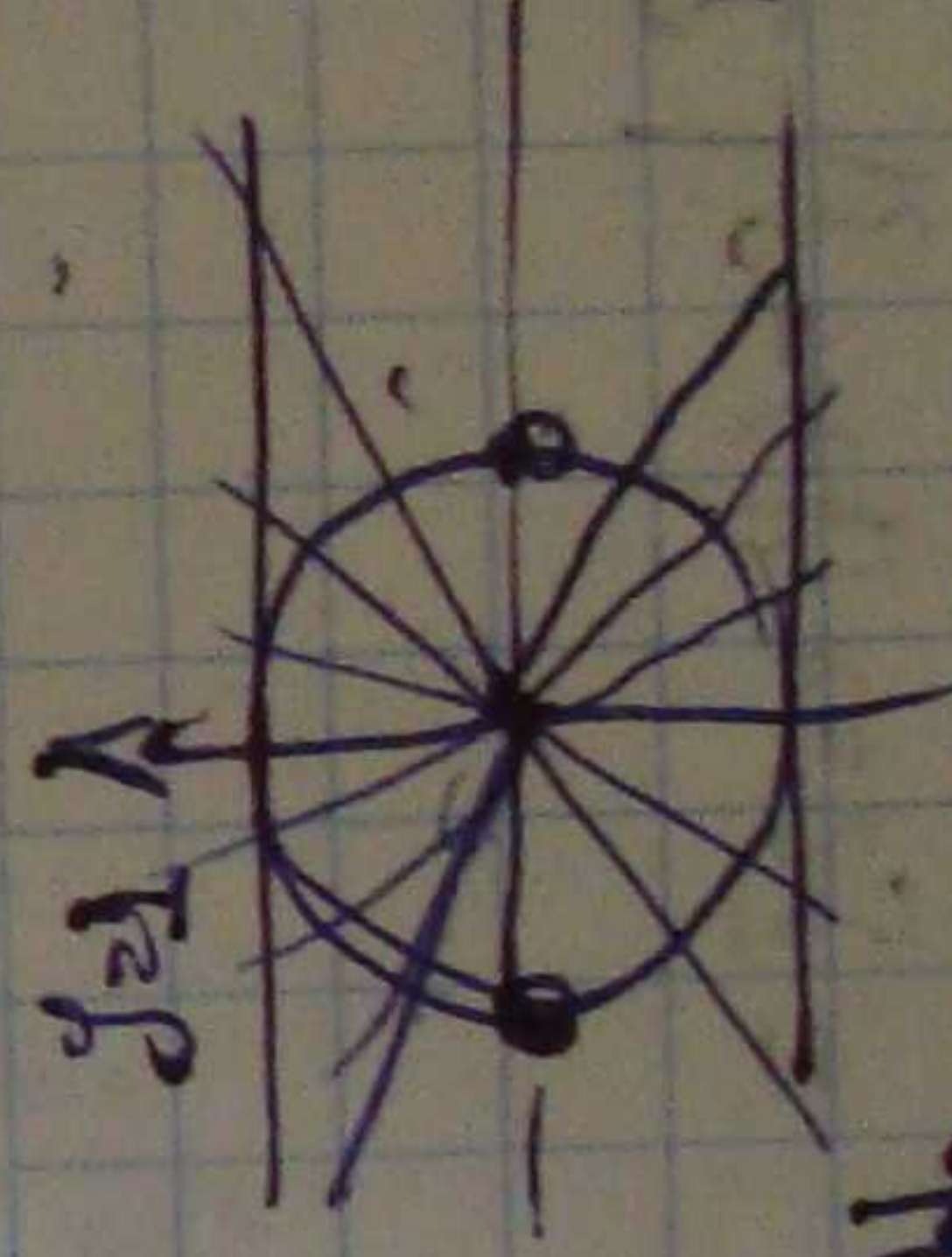
$$k \in \mathbb{Z}$$



$$\text{If } \operatorname{ctg} x < 0.$$

$$x \in \left(\frac{\pi}{2}, \pi + \pi k; \pi(k+1) \right)$$

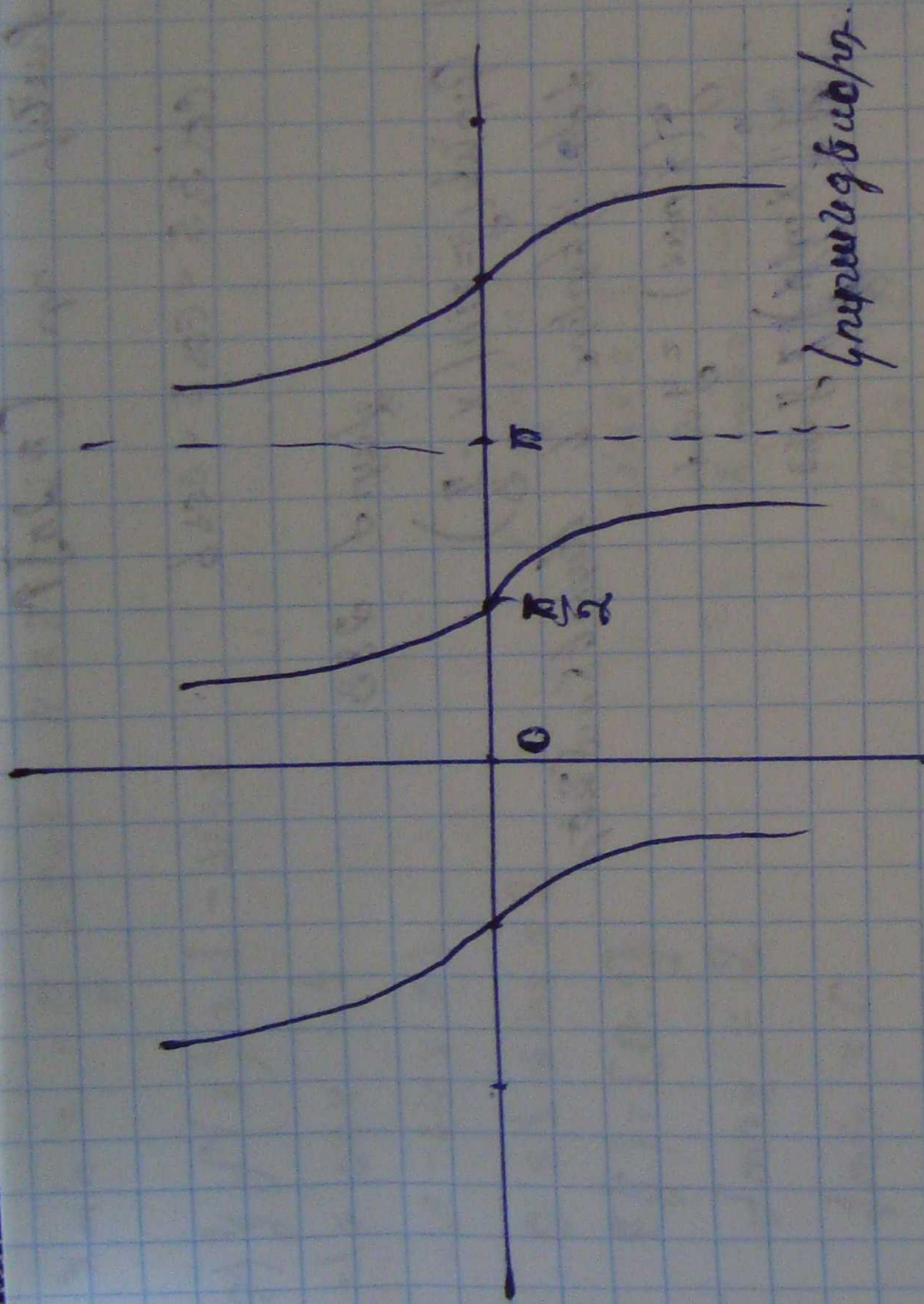
$$k \in \mathbb{Z}$$



$$x_2 > x_1 \\ \operatorname{ctg} x_2 < \operatorname{ctg} x_1$$

В. Син. Ф.?

$$x \in \left(\pi k; \pi(k+1) \right)$$



$$x = \pi k, k \in \mathbb{Z}$$

հարաբերակցություն

$$\begin{cases} x \rightarrow 0 \\ \operatorname{ctg} x \rightarrow +\infty \end{cases}$$

հարաբերակցություն

$$\cos 3,5; \cos 6; \cos 5 \quad \text{quasi. ածական հարաբերակցություն}$$

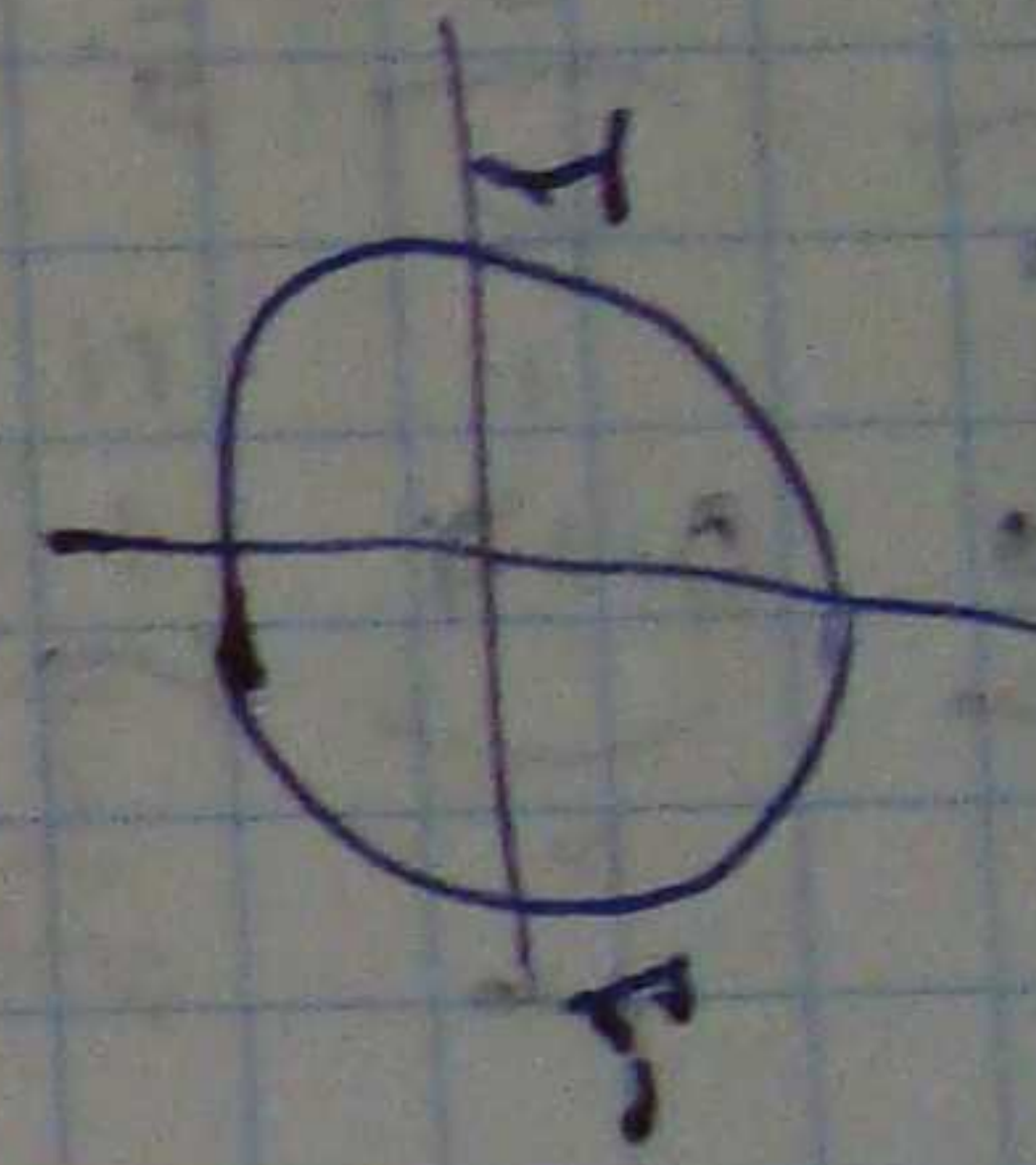
$$(\cos 6; \cos 5; \cos 3,5)$$

$$\pi < 3,5 < \frac{3\pi}{2}$$

$$\frac{3\pi}{2} < 5 < 2\pi$$

$$\frac{3\pi}{2} < 6 < 2\pi$$

$$\pi < 5 < 6 < 2\pi$$



Функция $\sin$ на $[\pi/2; 2\pi]$

$$\cos 3,5 < \cos 5 < \cos 6$$

Задача 283.

а) Найти $y = \sin(x - \frac{\pi}{3})$

Знайти экстремумы функции

$$y(x_{\max}) = y_{\max}$$

$$y(x_{\min}) = y_{\min}$$

$y_{\max}$

$$E(\sin x) \in [-1; 1]$$

$$y_{\max} = 1$$

$$\sin(x - \frac{\pi}{3}) = 1$$

$$x - \frac{\pi}{3} = \frac{\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}$$

$$x_{\max} = \frac{\pi}{2} - \frac{\pi}{3} + 2\pi k = \frac{5\pi}{6} + 2\pi k$$

$$y_{\min} = -1$$

$$\sin(x - \frac{\pi}{3}) = -1$$

$$\sin(x - \frac{\pi}{3}) = -1 \Rightarrow x - \frac{\pi}{3} = -\frac{\pi}{2} + 2\pi k$$

$$x_{\min} = -\frac{\pi}{6} + 2\pi k; k \in \mathbb{Z}$$

$$g) y f(x) = 1 - \sin x$$

$$-1 \leq \sin x \leq 1$$

$$-1 \leq -\sin x \leq 1$$

$$0 \leq 1 - \sin x \leq 2$$

$$E(f) = [0; 2]$$

$$f_{\max} = 2$$

$$f_{\min} = 0$$

$$1 - \sin x = 2$$

$$\sin x = -1$$

$$x_{\max} = -\frac{\pi}{2} + 2\pi k \quad k \in \mathbb{Z}$$

$$f_{\min} = 0$$

$$1 - \sin x = 0$$

$$\sin x = 1$$

$$x_{\min} = \frac{\pi}{2} + 2\pi k \quad k \in \mathbb{Z}$$

284.

$$n) y = 4 + 5 \sin x$$

$$y_{\min} \quad y_{\max}$$

$$-1 \leq \sin x \leq 1$$

$$-5 \leq 5 \sin x \leq 5$$

$$-1 \leq y + 5 \sin x \leq 9$$

$$-1 \leq y \leq 9$$

$$y_{\min} = 9$$

$$y_{\max} = -1$$

285/

$$P y = 2 \sin^2 x - 3 \sin x - 5$$

$$= 2 \left(\sin^2 x - \frac{3}{2} \sin x - \frac{5}{2} \right)$$

$$= 2 \left(\sin^2 x - 2 \cdot \frac{3}{4} \sin x + \frac{9}{16} - \frac{9}{16} - \frac{5}{2} \right)$$

$$= 2 \left(\sin x - \frac{3}{4} \right)^2 - \frac{49}{16}$$

$$= 2 \left(\sin x - \frac{3}{4} \right)^2 - \frac{49}{8}$$

$$2 \left(-\frac{3}{4} \right)^2 - \frac{49}{8}$$

$$\sin x = -\frac{1}{2}$$

$$y_{\max} = 2 \left(-\frac{1}{4} \right)^2 - \frac{49}{8}$$

$$y \sin x = 1$$

$$y_{\min} = 2 \left(1 - \frac{3}{4} \right)^2 - \frac{49}{8} = \frac{1}{8} - \frac{49}{8} = -6$$

$$q) y = 3 \cos^2 x - 6 \cos x = 3 (\cos^2 x - 2 \cos x) = 3 ((\cos x - 1)^2 - 1) = 3 (\cos x - 1)^2 - 3$$

$$\cos x = -1$$

$$y_{\max} = 3 \cdot 4 - 3 = 9$$

$$\cos x = 1$$

$$y_{\min} = -3$$

Wurde für

289-295; 292-296; 298-300.

w) $f(x) = \cos(x^2 - 2x + 1)$ $x \in [0; 1]$

$$x^2 - 2x + 1 = (x-1)^2$$

$$f(x) = \cos(x-1)^2$$

$$d > 0$$

$$\div x; y; z.$$

$$\div x+1; y+1; z+1$$

$$x+y+z=42$$

$$2y =$$

$$\begin{cases} a_1 + d = 14 \end{cases}$$

$$15^2 = (a_1 + 1)(14 + 21 + d)$$

$$d = 14 - a_1$$

$$(a_1 + 1)(49 - a_1) = 225$$

$$49a_1 - a_1^2 + 49 - a_1 = 225$$

$$a_1^2 - 48a_1 + 176 = 0$$

$$\frac{d}{4}, 24^2 - 176 = 576 - 176 = 400$$

$$a_1, 2 \quad 24 \pm 20 = 44$$

$$a_1 = 4; d = 10$$

Für $d > 0$

$$\div a_1; a_1 + d; a_1 + 2d.$$

$$\div a_1 + 1; a_1 + 1 + d; a_1 + 21 + 2d.$$

$$\begin{cases} a_1 + a_1 + d + a_1 + 2d = 42 \\ a_1 + 1 + d = \sqrt{(a_1 + 1)(a_1 + 21 + 2d)} \end{cases}$$

$$\begin{cases} a_1 + d = 14 \end{cases}$$

$$\begin{cases} (a_1 + 1)(d + 35) = 225 \end{cases}$$

$$\begin{cases} d = -30 \\ d = 10 \end{cases} \quad \text{für } d > 0, \text{ nur}$$

unterschied 289 - 295; 292 - 296; 298 - 300.

$$u) f(x) = \cos(x^2 - 2x + 1) \quad x \in [0; 1]$$

$$x^2 - 2x + 1 = (x - 1)^2$$

$$f(x) = \cos(x - 1)^2$$

$$d > 0$$

$$\div x; y; z.$$

$$\div x+1; y+1; z+1$$

$$x+y+z=42$$

$$2y =$$

$$\begin{cases} a_1 + d = 14 \end{cases}$$

$$15^2 = (a_1 + 1)(14 + 21 + d)$$

$$d = 14 - a_1$$

$$(a_1 + 1)(49 - a_1) = 225$$

$$49a_1 - a_1^2 + 49 - a_1 = 225$$

$$a_1^2 - 48a_1 + 176 = 0$$

$$\frac{d}{4}, 24^2 - 176 = 576 - 176 = 400$$

$$a_1; 2 \quad 24 \pm 20 = 44$$

$$a_1 = 44; d = 10$$

Zusatz 290

$$\div a_1; a_1 + d; a_1 + 2d.$$

$$\div a_1 + 1; a_1 + 1 + d; a_1 + 21 + 2d.$$

$$\begin{cases} a_1 + a_1 + d + a_1 + 2d = 42 \end{cases}$$

$$\begin{cases} a_1 + 1 + d = \sqrt{(a_1 + 1)(a_1 + 2d + 2)} \end{cases}$$

$$\begin{cases} a_1 + d = 14. \end{cases}$$

$$\begin{cases} (a_1 + 1)(d + 35) = 225 \end{cases}$$

$$\begin{cases} d = -30 \\ d = 10 \end{cases} \quad \text{für } d > 0, \text{ nur}$$

$$a_1 + 2d + 21 = 4 + 20 + 21 = 45$$

Sum: 45

$$24. \quad \div 4; 14; 28$$

$$\div 5; 15; 45$$

Faktor 29

$$d > 0$$

$$\div x; y; z. \quad x + y + z = 30$$

$$\div x=5; y=4; z$$

$$\begin{cases} x + y + z = 30 \\ 2y = x + z \\ (y-4)^2 = z(x-5) \end{cases} \Rightarrow \begin{cases} 3y = 30 \\ 2y = x + z \\ (y-4)^2 = z(x-5) \end{cases}$$

$$\begin{cases} y = 10 \\ x + z = 20 \\ z(x-5) = 36 \end{cases} \Rightarrow \begin{cases} x = 20 - z \\ z(15 - z) = 36 \\ y = 10 \end{cases}$$

$$\begin{cases} x = 20 - z \\ y = 10 \\ z^2 - 15z + 36 = 0 \end{cases}$$

$$\Delta = 225 - 144 = 81$$

$$z_{1,2} = \frac{15 \pm 9}{2} = 12$$

$$a_1 + 2d + 21 = 4 + 20 + 21 = 45$$

Sum: 45

$$26. \div 4; 14; 28$$

$$\div 5; 15; 45$$

Fulcrum 29

$$d = 0$$

$$\div x; y; z. \quad x + y + z = 30$$

$$\div x=5; y=4; z$$

$$\begin{cases} x + y + z = 30 \\ 2y = x + z \\ (y-4)^2 = z(x-5) \end{cases} \Rightarrow \begin{cases} 3y = 30 \\ 2y = x + z \\ (y-4)^2 = z(x-5) \end{cases}$$

$$\Rightarrow \begin{cases} y = 10 \\ x + z = 20 \\ z(x-5) = 36 \end{cases} \Rightarrow \begin{cases} x = 20 - z \\ z(15 - z) = 36 \\ y = 10 \end{cases}$$

$$\Rightarrow \begin{cases} x = 20 - z \\ y = 10 \\ z^2 - 15z + 36 = 0 \end{cases}$$

$$D = 225 - 144 = 81$$

$$z_{1,2} = \frac{15 \pm 9}{2} = 12$$

Решить при $d > 0$, найти $z > y$, найти $\arg z = 10$

найдем $z = 12 \Rightarrow$

$$z = \begin{cases} y = 10 \\ z = 12 \\ x = 8 \end{cases}$$

Найдем: $\div 8; 10; 12$

$\div 3; 6; 12$

найдем 294.

Найдем, найдем, найдем (294-295)

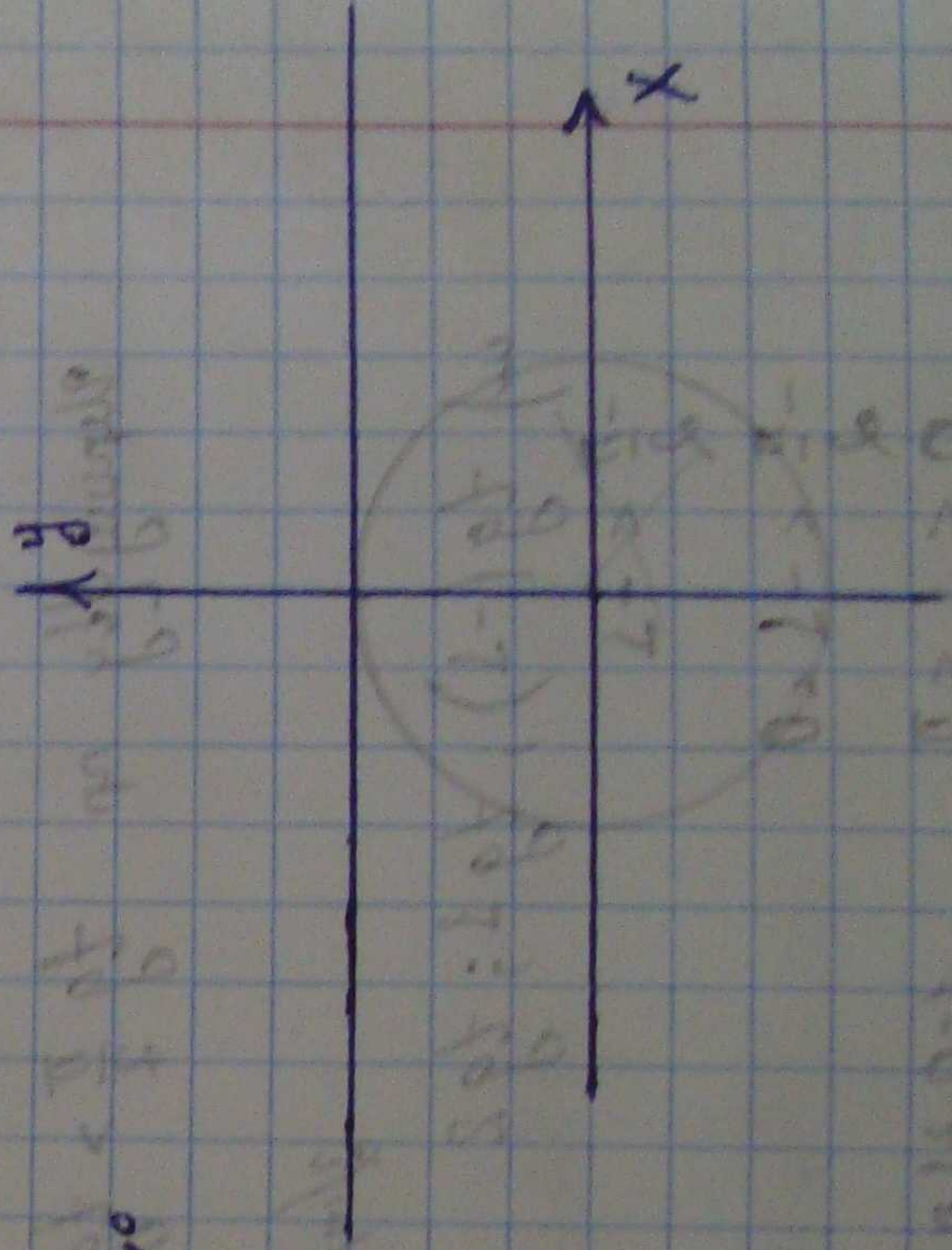
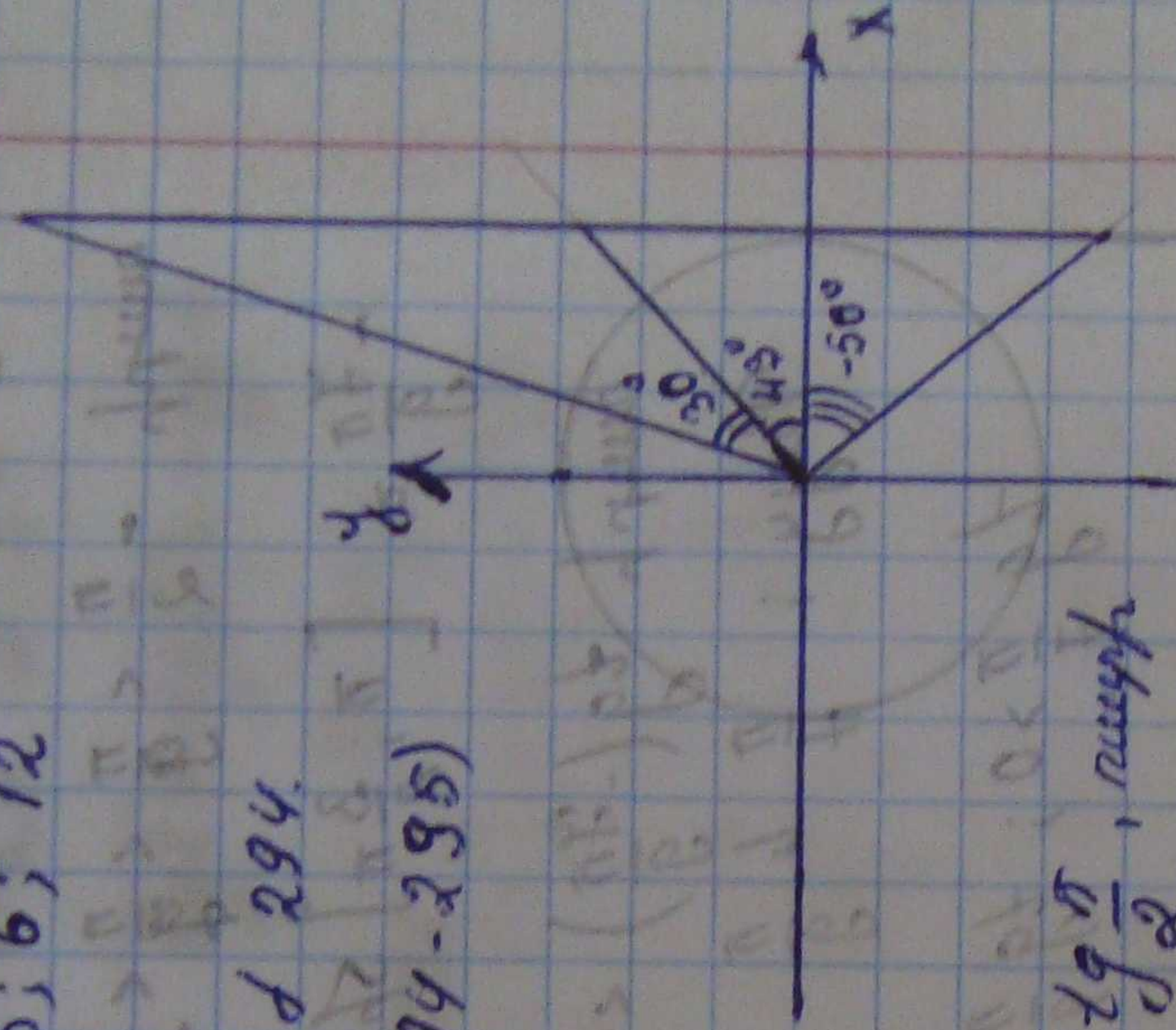
а) $\arg 43^\circ; \arg 73^\circ; \arg(-50^\circ)$

$\arg -0 \left[-\frac{\pi}{2}; \frac{\pi}{2} \right] \pi$, найдем

$-\frac{\pi}{2} < \arg(-50^\circ) < \arg 43^\circ < \arg 73^\circ < \arg \frac{\pi}{2}$, найдем

$\arg 73^\circ > \arg 43^\circ > \arg(-50^\circ)$

Р) $\arg 72^\circ; \arg 13^\circ; \arg 104^\circ$



$$(2 > 1 > -1) \quad \text{tg} 2 > \text{tg} 1 > \text{tg} (-1)$$

$$n) \quad \text{tg} 2, \text{tg} 4, \text{tg} 2, 5$$

$$\frac{\pi}{2} < 2 < \pi$$

$$\pi < 4 < \frac{3}{2}\pi$$

$$\frac{\pi}{2} < 2, 5 < \pi$$

$$\text{Правильно } \arctan \left[\frac{\pi}{2}; \frac{3}{2}\pi \right]$$

$$\text{нужно } \text{tg} 2 > \text{tg} 2, 5 > \text{tg} 4$$

$$\text{Значит } \arctan(f) \in \text{дуга } 296$$

$$\text{мы } f(x) = \frac{1}{\text{tg} x} = \text{ctg} x = \frac{\cos x}{\sin x}$$

$$\sin x \neq 0$$

$$x$$

$$\frac{\pi}{2} + \pi k, -\frac{\pi}{2} + \pi k$$

$$\sin x \in \left[-1; \frac{\pi}{2} + \pi k \right) \cup \left(\frac{\pi}{2} + \pi k; 1 \right]$$

$$\arctan(f) = \left[-1; \frac{\pi}{2} + \pi k \right) \cup \left(\frac{\pi}{2} + \pi k; 1 \right]$$

$$P) \quad f(x) = \frac{2}{\text{ctg} x} = \frac{2 \sin x}{\cos x}$$

$$\cos x = 0, \text{ тогда } x = \frac{\pi}{2} + \pi k$$

$$\text{то } \arctan(f) = \left[-1; \frac{\pi}{2} + \pi k \right) \cup \left(\frac{\pi}{2} + \pi k; 1 \right]$$

$$(2 > 1 > -1) \quad \text{tg} 2 > \text{tg} 1 > \text{tg}(-1)$$

$$n) \quad \text{tg} 2, \text{tg} 4, \text{tg} 2, 5$$

$$\frac{\pi}{2} < 2 < \pi < \frac{3\pi}{2}$$

$$\pi < 4 < \frac{3\pi}{2}$$

$$\frac{\pi}{2} < 2, 5 < \pi$$

$$\text{quadrant } \phi(f) = \text{quadrant } 296$$

$$iii) \quad f(x) = \frac{1}{\text{tg} x} = \text{ctg} x = \frac{\cos x}{\sin x}$$

$$\sin x \neq 0$$

$$\frac{\pi}{2} + \pi k, -\frac{\pi}{2} + \pi k$$

$$\sin x \in [-1; \frac{\pi}{2} + \pi k) \cup (\frac{\pi}{2} + \pi k; 1]$$

$$\phi(f) = [-1; \frac{\pi}{2} + \pi k) \cup (\frac{\pi}{2} + \pi k; 1]$$

$$ii) \quad f(x) = \frac{2}{\text{ctg} x} = \frac{2 \sin x}{\cos x}$$

$$\cos x = 0, \text{tg} x = \frac{\pi}{2} + \pi k$$

$$iv) \quad \phi(f) = [-1; \frac{\pi}{2} + \pi k) \cup (\frac{\pi}{2} + \pi k; 1]$$

$$x \text{tg} 0 - 1 = (x) f(x)$$

$$0 > x \text{tg} 0$$

$$\text{function } \text{tg} x \in [\frac{\pi}{2}; \frac{3\pi}{2}]$$

$$\text{number } \text{tg} 2 > \text{tg} 2, 5 > \text{tg} 4$$

$$0 < x \text{tg} 0$$

$$\text{quadrant } \text{tg} x \in [\frac{\pi}{2}; \frac{3\pi}{2}]$$

$$\frac{1}{x \text{tg} 0 - 1} = (x) f(x)$$

$$0 > x \text{tg} 0$$

$$\text{quadrant } \text{tg} x \in [\frac{\pi}{2}; \frac{3\pi}{2}]$$

$$ii) \quad f(x) = \frac{1}{x \text{tg} 0 - 1}$$

$$0 < x \text{tg} 0$$

$$\text{quadrant } \text{tg} x \in [\frac{\pi}{2}; \frac{3\pi}{2}]$$

$$0 < 12$$

$$0 < 7$$

$$0 < 8$$

$$0 < 20 + 0 + 7 = 27$$

$$g) f(x) = \sqrt{1 - \cot^2 x}$$

$$\cot x \leq 0$$

$$\omega(f) = \cot x \in \left(\left[\frac{\pi}{2} + \pi k; \pi + \pi k \right) \cup \left[\frac{3}{2}\pi + \pi k; 2\pi + \pi k \right) \right)$$

$$a) f(x) = \sqrt{\tan x}$$

$$\tan x > 0$$

$$\omega(f) = \tan x \in \left(\left[\pi k; \frac{\pi}{2} + \pi k \right) \cup \left[\pi + \pi k; \frac{3}{2}\pi + \pi k \right) \right)$$

$$b) f(x) = \frac{1}{\sqrt{1 - \tan^2 x}}$$

$$\tan x < 0$$

$$\omega(f) = \tan x \in \left(\left(\frac{\pi}{2} + \pi k; \pi + \pi k \right) \cup \left(\frac{3}{2}\pi + \pi k; 2\pi + \pi k \right) \right)$$

$$v) f(x) = \frac{1}{\sqrt{\cot^2 x}}$$

$$\cot x > 0$$

$$\omega(f) = \cot x \in \left(\left(\pi k; \frac{\pi}{2} + \pi k \right) \cup \left(\pi + \pi k; \frac{3}{2}\pi + \pi k \right) \right)$$

Решение 299

$$S_n = \frac{2a_1 + (n-1)d}{2} \cdot n$$

$$a_1 = 15$$

$$d = -1$$

$$S_n = 84$$

$$n = ?$$

$$\frac{n(30 + n + 1)}{2} = 84$$

$$3a_n - n^2 = 168$$

$$n^2 - 31n + 168 = 0$$

$$b = 961 + 168 = 1129$$

$$48 \sqrt{1129} = 2$$

$$\begin{array}{r} 31 \\ + 31 \\ \hline 62 \\ 93 \\ \hline 961 \\ - 168 \\ \hline 793 \end{array}$$

(Faktor 300)

$$S_n = \frac{2a_1 + (n-1)d}{2} \cdot n$$

$$2S_n = (2a_1 + (n-1)d) \cdot n$$

$$n = \frac{2S_n}{2a_1 + (n-1)d} = \frac{2 \cdot 84}{30 + 1 \cdot n} = \frac{168}{31 - n}$$

Faktor 300

$$a_1 = 50$$

$$d = 2$$

$$S_n = 1470$$

$$n = ?$$

$$S_n = \frac{2a_1 + (n-1)d}{2} \cdot n$$

$$S_n = a_1 + n - 1$$

$$n = S_n - a_1 + 1 = 1470 - 50 + 1 =$$

$$= 1421$$

Temp: 1421 - 121

values

a principal value $\arcsin a \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Specially

$$-\frac{\pi}{2} \leq \arcsin a \leq \frac{\pi}{2}$$

$$\sin(\arcsin a) = a \quad |a| \leq 1$$

$$\sin(\arcsin(-a)) = -\arcsin a$$

$$-\frac{\pi}{2} \leq -\arcsin a \leq \frac{\pi}{2}$$

$$\sin(-\arcsin a) = -\sin(\arcsin a) = -a$$

$$\arcsin 0 = 0^\circ$$

$$\arcsin \frac{1}{2} = 30^\circ$$

$$\arcsin \frac{\sqrt{2}}{2} = 45^\circ$$

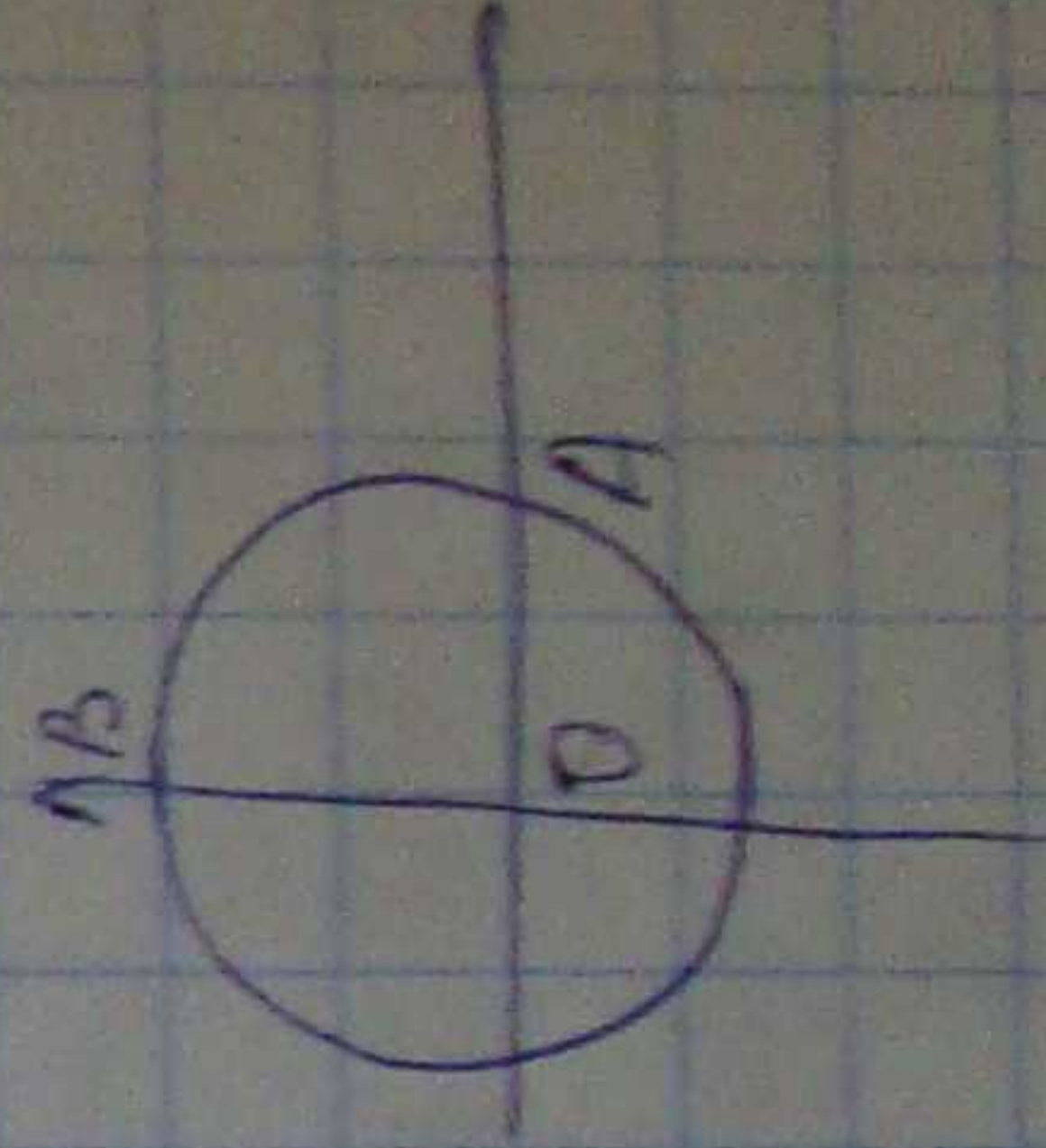
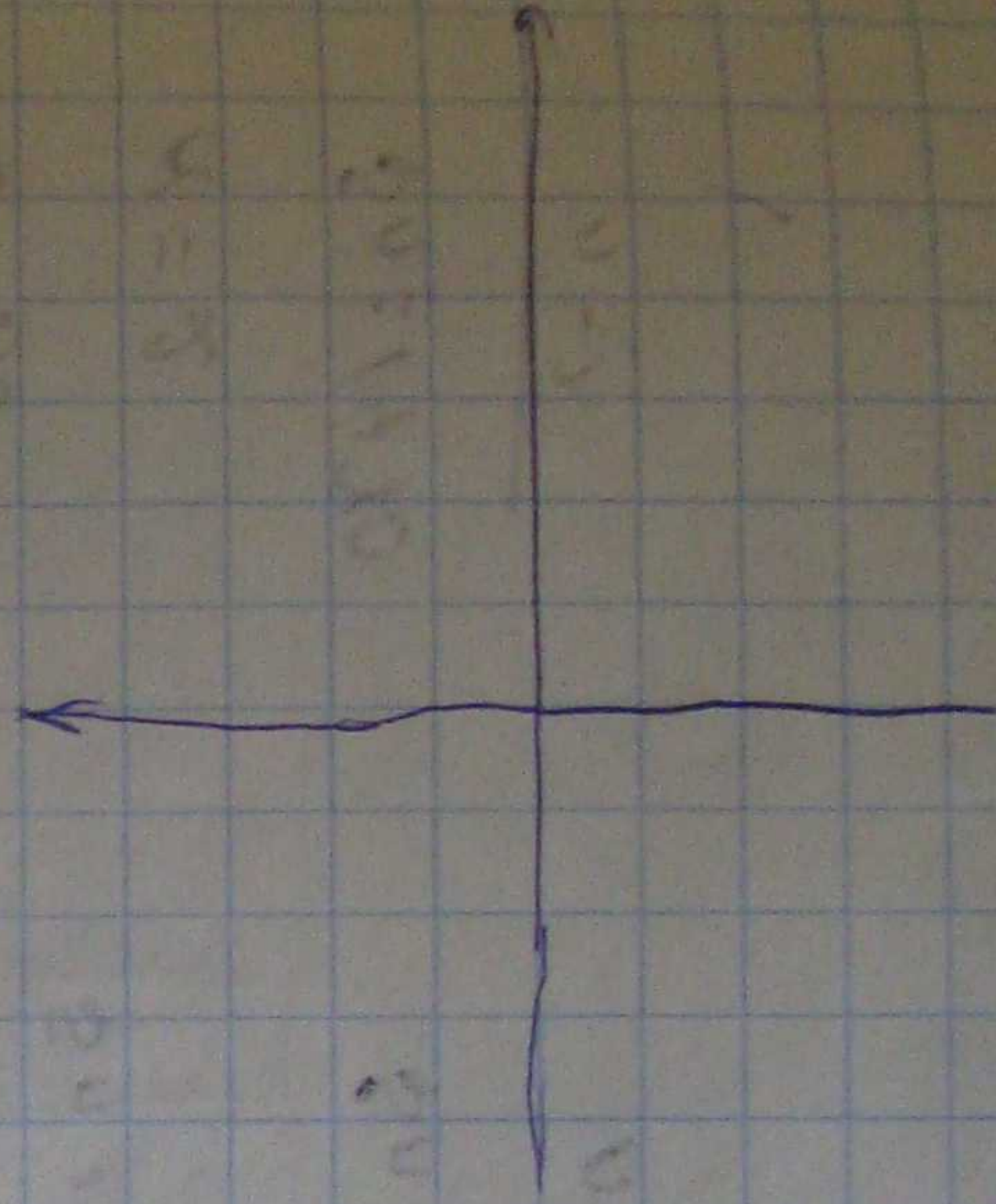
$$\arcsin \frac{\sqrt{3}}{2} = 60^\circ$$

$$\arcsin 1 = 90^\circ$$

$$\sin x = 1$$

$$x = \frac{\pi}{2} + 2\pi k$$

$$\sin x = -1$$



$$0 \leq \theta < 2\pi$$
$$-2\pi < \theta \leq 0$$
$$0 \leq \theta < 2\pi$$

$$x = \frac{\pi}{2} + 2\pi k$$

$$\sin x = 0$$

$$x = \pi k$$

$$\sin \alpha \cos \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$2. \cos x = a$$

$$\text{uf } |a| > 1 \quad x \in \phi$$

$$1) |a| \leq 1$$

$$x = \pm \alpha + 2\pi k$$

$$\cos x = a \quad |a| \leq 1.$$

$$x = \pm \arccos a + 2\pi k; k \in \mathbb{Z}$$

uf a pyff $\arccos$ - $\{ \text{pyffus } [0; \pi] \}$,

uf a pyff $\arccos$ (a pyff $\arccos$), $\arccos$

$$\cos - a = \pm a:$$

$$\cos(\arccos a) = a$$

$$\text{huyff } \arccos(-a) = \pi - \arccos a$$

$$0 \leq \arccos a \leq \pi$$

$$-\pi \leq -\arccos a \leq 0.$$

$$0 \leq \pi - \arccos a \leq \pi$$

$$\cos(\pi - \arccos a) = -\cos(\arccos a) = -a.$$



$$\arccos\left(-\frac{1}{2}\right) = \pi - \arccos\frac{1}{2} =$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$



$$\arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

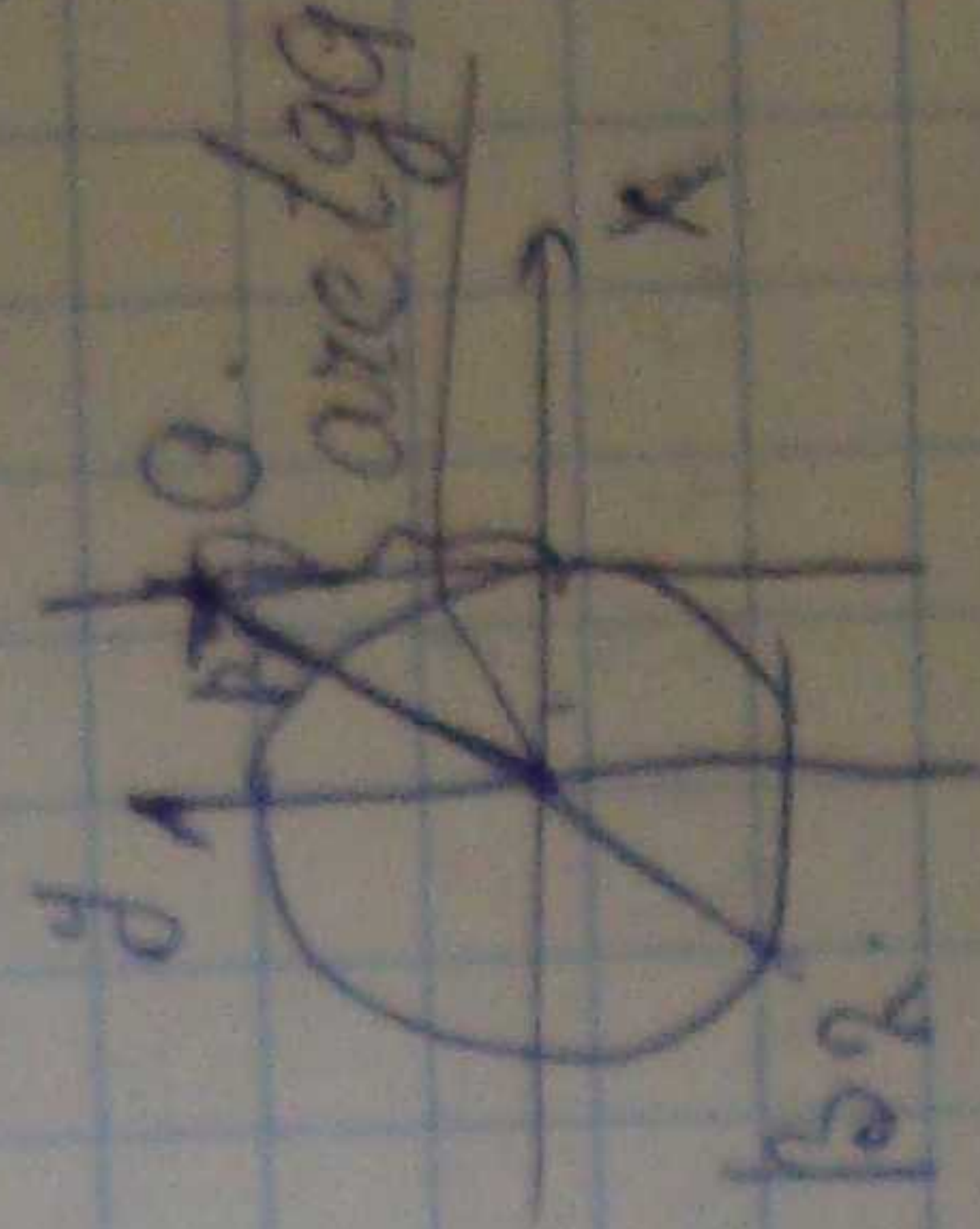
$$\arccos\frac{1}{2} = \frac{\pi}{3}$$

$$\arccos 0 = \frac{\pi}{2}$$

$$3. \cos x = 0$$

$$x = \frac{\pi}{2} + 2\pi k; k \in \mathbb{Z}$$

$$x = \arctg a + \pi k$$



$$4. \operatorname{tg} x = a$$

$$a \in \mathbb{R}$$

$$a > 0$$

$$x = \alpha + \pi k$$

$$-\frac{\pi}{2} < \arctg a < \frac{\pi}{2}$$

$$\operatorname{tg}(\arctg a) = a \quad a \in \mathbb{R}$$

$$\operatorname{tg}(\arctg(-a)) = -\arctg a$$

$$-\arctg a \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\operatorname{tg}(-\operatorname{arctg} a) = -\operatorname{tg}(\operatorname{arctg} a) = -a.$$

$$\operatorname{arctg} 0 = 0$$

$$\operatorname{tg} x = 0$$

$$\operatorname{arctg} \frac{\sqrt{3}}{3} = \frac{\pi}{6}$$

$$x = \pi k; k \in \mathbb{Z}.$$

$$\operatorname{arctg} 1 = \frac{\pi}{4}$$

$$\operatorname{arctg} \sqrt{3} = \frac{\pi}{3}$$

$$4. \operatorname{ctg} x = a$$

$$x = \operatorname{arccotg} a + \pi k; k \in \mathbb{Z}$$

$$0 < \operatorname{arccotg} a < \pi$$

$$\operatorname{ctg}(\operatorname{arctg} a) = a.$$

hump.

$$\operatorname{arccotg}(-a) = \pi - \operatorname{arccotg} a.$$

$$\operatorname{ctg} \operatorname{arccotg} \left(\frac{\pi}{2} + \pi k \right)$$

200 p 4m 303

$$u) \arcsin\left(-\frac{1}{2}\right) + \arccos\left(-\frac{\sqrt{3}}{2}\right) =$$

$$= -30^\circ + 30^\circ = 0.$$

$$p) \arcsin\left(-\frac{1}{2}\right) + \arccos(-1) =$$

$$= -\frac{3}{2}\pi + \pi = \frac{5\pi}{2}.$$

$$q) \arcsin\left(-\frac{\sqrt{2}}{2}\right) + \arccos\frac{\sqrt{3}}{2} =$$

$$= -\arcsin\frac{\sqrt{2}}{2} + \frac{\pi}{6} =$$

$$= -\frac{\pi}{4} + \frac{\pi}{6} = \frac{-3\pi + 2\pi}{12} = -\frac{\pi}{12}.$$

2/4m 304

$$\arcsin\frac{1}{3} =$$

$$b) \arcsin\frac{1-\sqrt{11}}{2} = \arcsin\left(\frac{1}{2} - \frac{\sqrt{11}}{2}\right).$$

$$\sin x = a.$$

$$y) \arcsin(\sqrt{4}-\sqrt{5}) =$$



матрица кватернионов $\arcsin \arccos$ 2-4

202 - 314, 316 - 323.

323, 324, 325 - 327:

2/урд 301

a	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\arcsin a$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$-\frac{\pi}{6}$	$-\frac{\pi}{4}$	$-\frac{\pi}{3}$	$-\frac{\pi}{2}$
$\arccos a$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	0	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

2/урд 302

$$\text{w) } \arcsin 0 + \arccos 1 = 0 + 0 = 0$$

$$\text{p) } \arcsin 1 + \arccos 0 = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$\text{q) } \arcsin \frac{\sqrt{2}}{2} + \arccos \frac{1}{2} = \frac{\pi}{4} + \frac{\pi}{3} = \frac{4\pi}{12}$$

$$\text{r) } \arccos \frac{1}{2} - \arccos \left(-\frac{1}{2}\right) = \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right) = \frac{2\pi}{3} - \pi = -\frac{\pi}{3}$$

б)

2/урд 303

$$\text{w) } \arcsin \left(-\frac{1}{2}\right) + \arccos \left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6} + \pi - \frac{\pi}{6} = \frac{2\pi}{3}$$

$$\text{p) } \arcsin(-1) + \arccos(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$$

$$\text{q) } \arccos \left(-\frac{\sqrt{3}}{2}\right) + \arcsin \frac{1}{2} = \frac{5\pi}{6} + \frac{\pi}{6} = \pi$$

$$7) \arcsin\left(-\frac{\sqrt{2}'}{2}\right) + \arccos\frac{\sqrt{3}'}{2} = -\frac{\pi}{4} + \frac{\pi}{6} = -\frac{\pi}{12}$$

Задание решено: 304

$$8) \arcsin\frac{1}{3} - \arccos\frac{\sqrt{3}-1}{2} = \arcsin\frac{1}{3} - \arccos\frac{\sqrt{3}-1}{2}$$

$$9) \arcsin\frac{\sqrt{3}+1}{2} - \arccos\frac{\sqrt{3}-1}{2} = \arcsin\frac{\sqrt{3}+1}{2} - \arccos\frac{\sqrt{3}-1}{2}$$

$$10) \arcsin\frac{1-\sqrt{11}'}{2} - \arccos\frac{\sqrt{7}-\sqrt{5}'}{2} = \arcsin\frac{1-\sqrt{11}'}{2} - \arccos\frac{\sqrt{7}-\sqrt{5}'}{2}$$

305

Задание решено: 305

$$11) \arcsin\frac{1}{3} + \arcsin\left(-\frac{1}{3}\right) = 0$$

$$\arcsin\frac{1}{3} - \arccos\left(-\frac{1}{3}\right) = \arcsin\frac{1}{3} - \arccos\frac{1}{3} = 0$$

$$12) \arcsin(\sqrt{3}-1) + \arcsin(1-\sqrt{3}) = 0$$

$$\arcsin(\sqrt{3}-1) - \arccos(\sqrt{3}-1) = 0$$

$$13) \arcsin(1-\sqrt{2}) + \arcsin\frac{1}{\sqrt{2}+1} = 0$$

$$\arcsin(1-\sqrt{2}) + \arcsin(\sqrt{2}-1) = \arcsin(1-\sqrt{2}) - \arccos(1-\sqrt{2})$$

$$w) \arcsin(\sin 5)$$

Звмуп рудлр;

$$\arccos \sqrt{5} - \pi \quad \arccos(\sqrt{5} - 5) - \pi \quad \arccos \frac{2}{3} - \pi$$

Звмуп рудлр;

$$w) \arccos \frac{2}{3} + \arccos\left(-\frac{2}{3}\right) = \pi$$

$$\arccos \frac{2}{3} + \arccos\left(-\frac{2}{3}\right) = \arccos \frac{2}{3} + \pi - \arccos \frac{2}{3} = \pi$$

$$p) \arccos(2 - \sqrt{5}) + \arccos \frac{1}{\sqrt{5} + 2} = \pi$$

$$\arccos(2 - \sqrt{5}) + \pi - \arccos(2 - \sqrt{5}) = \pi$$

Звмуп

$$\arccos\left(\frac{\cos(\frac{\pi}{5})}{a}\right) = \alpha$$

$$\arccos(\cos(-1)) = 1$$

$$\cos \alpha = a = \cos(-1) = \pi - \cos 1$$

$$\alpha = 1$$

Звмуп

$$w) \arcsin\left(\cos \frac{\pi}{5}\right) = \arcsin\left(\sin \frac{3\pi}{10}\right) = \frac{3\pi}{10}$$

2/10/15

a	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$
$\arctg a$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$-\frac{\pi}{6}$	$-\frac{\pi}{4}$	$-\frac{\pi}{3}$
$\operatorname{arctg} a$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$

2/10/15

$$u) \arctg 1 - \arctg \sqrt{3} = \frac{\pi}{4} - \frac{\pi}{3} = \frac{\pi}{12}$$

$$p) \arctg\left(\frac{\sqrt{3}}{3}\right) + \arctg(-\sqrt{3}) = \frac{\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{6}$$

$$q) \operatorname{arctg} 0 + \operatorname{arctg}(-\sqrt{3}) = \frac{\pi}{2} + \frac{5\pi}{6} = \frac{4\pi}{3}$$

$$r) \operatorname{arctg}(-1) - \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = \frac{3\pi}{4} + \frac{\pi}{6} = \frac{11\pi}{12}$$

$$b) \arctg \frac{\sqrt{3}}{3} - \arctg 0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

$$y) \arctg(-\sqrt{3}) - \arctg 1 = -\frac{\pi}{3} - \frac{\pi}{4} = -\frac{7\pi}{12}$$

2/10/15

$$u) \arctg 7 + \arctg(-7) = 0$$

$$\arctg 7 - \arctg 7 = 0$$

$$p) \operatorname{arctg}(\sqrt{5}-5) + \operatorname{arctg}(5-\sqrt{5}) = \pi$$

$$\star \arctan(\sqrt{5}-5) + \pi - \arctan(\sqrt{5}-5) = \pi$$

$$n) \arctan(1-\sqrt{7}) + \arctan\left(\frac{6}{\sqrt{7}+1}\right) = 0$$

$$\arctan(1-\sqrt{7}) - \arctan(6(1-\sqrt{7}))$$

2/4/20 323

$$w) \frac{x-2}{4} + g \cdot \sqrt{\frac{x-2}{4}} = 10$$

$$\left(\sqrt{\frac{x-2}{4}}\right)^2 + g \sqrt{\frac{x-2}{4}} - 10 = 0$$

$$g = \sqrt{\frac{x-2}{4}}$$

$$g^2 + gy - 10 = 0$$

$$20 = 81 - 40 = 41$$

$$\pi - \arctan(\sqrt{5}-5) + \pi - \arctan(\sqrt{5}-5) = \pi$$

$$n) \arctan(1-\sqrt{4}) + \arctan\left(\frac{6}{\sqrt{4}+1}\right) = 0$$

$$\arctan(1-\sqrt{4}) - \arctan(6(1-\sqrt{4}))$$

2/2/2023

$$\text{w/ } \frac{x-2}{4} + 9 \cdot \sqrt{\frac{x-2}{4}} = 10$$

$$\left(\sqrt{\frac{x-2}{4}}\right)^2 + 9\sqrt{\frac{x-2}{4}} - 10 = 0$$

$$y = \sqrt{\frac{x-2}{4}}$$

$$y^2 + 9y - 10 = 0$$

$$y = 81 - 40 = 41$$

$$\sin x = a$$

$$|a| \leq 1$$

$$x = (-1)^k \arcsin a + \pi k; k \in \mathbb{Z}$$

$$-\frac{\pi}{2} \leq \arcsin a \leq \frac{\pi}{2}$$

$$\sin(\arcsin a) = a$$

$$|a| \leq 1$$

$$\arcsin(-a) = -\arcsin a$$

$$\arcsin(\sin x) = x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\arcsin(\sin x) = x - \pi k$$

$$k = 2n \quad \arcsin(\sin x) = x - 2\pi n$$

$$k = 2n + 1$$

$$\arcsin(\sin x) = (2n + 1)\pi - x$$

$$\cos x = a$$

$$|a| \leq 1$$

$$x = \pm \arccos a + 2\pi n$$

$$0 \leq \arccos a \leq \pi$$

$$\cos(\arccos a) = a$$

$$1. \arccos(-a) = \pi - \arccos a \quad |a| \leq 1$$

$$2. \arccos(\cos x) = x; x \in [0; \pi]$$

$$\arccos x \notin [0; \pi]$$

$$\arccos(\cos x) = x - \pi k \quad k \in [0; \pi]$$

$$\text{где } k = 2n.$$

$$\arccos(\cos x) = x - x - 2\pi n$$

$$\arccos(\cos x) = (k+1)\pi - k$$

$$\operatorname{tg} x = a \quad a \in \mathbb{R}$$

$$x = \operatorname{arctg} a + \pi k$$

$$-\frac{\pi}{2} < \operatorname{arctg} a < \frac{\pi}{2}$$

$$1. \operatorname{tg}(\operatorname{arctg} a) = a \quad 1. \operatorname{arctg}(-a) = -\operatorname{arctg} a$$

$$2. \operatorname{arctg}(\operatorname{tg} x) = x, \text{ где } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\text{где } x \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \text{ any } a$$

$$\operatorname{arctg}(\operatorname{tg} x) = x - \pi k$$

2/4/20, 325

$$9) 2 \sin 4x = \sqrt{3}$$

$$\sin 4x = \frac{\sqrt{3}}{2} < 1$$

$$4x = \frac{\pi}{6}$$

$$x = \frac{\pi}{24}$$

$$4x = (-1)^k \arcsin \frac{\sqrt{3}}{2} + \pi k$$

$$4x = (-1)^k \frac{\pi}{6} + \pi k$$

$$x = \frac{(-1)^k \pi}{12} + \frac{\pi}{4} k \quad k \in \mathbb{Z}$$

$$1) 2 \sin \frac{x}{5} = -\sqrt{2}$$

$$\sin \frac{x}{5} = -\frac{\sqrt{2}}{2}$$

$$\frac{x}{5} = (-1)^k \arcsin \left(-\frac{\sqrt{2}}{2} \right) + \pi k$$

$$\frac{x}{5} = (-1)^k \cdot 5 \cdot (-1) \cdot \frac{\pi}{4} + 5\pi k = (-1)^{k+1} \cdot \frac{5\pi}{4} + 5\pi k$$

326

$$7) \sqrt{2} \cos 2x - 1 = 0.$$

$$\sqrt{2} \cos 2x = 1.$$

$$\cos 2x = \frac{\sqrt{2}}{2}.$$

$$2x = \pm \arccos \frac{\sqrt{2}}{2} + 2\pi k$$

$$2x = \pm \frac{\pi}{4} + 2\pi k.$$

$$x = \pm \frac{\pi}{8} + \pi k.$$

$$9) 2 \cos 4x + \sqrt{3} = 0.$$

$$\cos 4x = -\frac{\sqrt{3}}{2}.$$

$$4x = \pm \arccos \left(-\frac{\sqrt{3}}{2} \right) + 2\pi k$$

$$4x = \pm \left(\pi - \frac{\pi}{6} \right) + 2\pi k \pm$$

$$x = \pm \frac{5\pi}{24} + \frac{\pi k}{2} \quad k \in \mathbb{Z}.$$

327

$$4) \sqrt{3} \operatorname{tg} \frac{x}{3} + 1 = 0$$

$$\operatorname{tg} \frac{x}{3} = -\frac{1}{\sqrt{3}}$$

$$\frac{x}{3} = \arctan\left(-\frac{1}{\sqrt{3}}\right) + \pi k \quad k \in \mathbb{Z}$$

$$\frac{x}{3} = -\frac{\pi}{6} + \pi k$$

$$x = -\frac{\pi}{2} + 3\pi k; \quad k \in \mathbb{Z}$$

2/супр. 329

$$p) \quad 2 \sin\left(3x + \frac{\pi}{4}\right) = -\sqrt{3}$$

$$\sin\left(3x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

$$3x + \frac{\pi}{4} = \pm (-1)^k \arcsin\left(-\frac{\sqrt{3}}{2}\right) + \pi k \quad k \in \mathbb{Z}$$

$$\neq 3x + \frac{\pi}{4} = (-1)^{k+1} \frac{\pi}{3} + \pi k$$

$$3x = -\frac{\pi}{4} + (-1)^{k+1} \frac{\pi}{3} + \pi k$$

$$x = -\frac{\pi}{12} + (-1)^{k+1} \frac{\pi}{9} + \frac{\pi k}{3} \quad k \in \mathbb{Z}$$

330

$$\cot\left(\frac{\pi}{4} - \frac{x}{2}\right) = 1.$$

$$\frac{\pi}{4} - \frac{x}{2} = \arccot 1 + \pi k$$

$$\frac{x}{2} = \frac{\pi}{4} - \frac{\pi}{4} - \pi k$$

$$\frac{x}{2} = -\pi k$$

$$x = 2\pi k$$

333.

$$\sqrt{3} \sin x - \sin 2x = 0$$

$$\sqrt{3} \sin x - 2 \sin x \cos x = 0.$$

$$\sin x (\sqrt{3} - 2 \cos x) = 0$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \pm \arccos \frac{\sqrt{3}}{2} + 2\pi k$$

$$\text{+1. } x = \pi k$$

$$\sin x = 0$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \pm \arccos \left(\frac{\sqrt{3}}{2} \right) + 2\pi k \Rightarrow$$

$$x = \pm \frac{\pi}{6} + 2\pi k$$

$$x = \pi k$$

k ∈ Z.

$$x = \pm \frac{\pi}{6} + 2\pi k.$$

330

$$\cot\left(\frac{\pi}{4} - \frac{x}{2}\right) = 1.$$

$$\frac{\pi}{4} - \frac{x}{2} = \arccot 1 + \pi k$$

$$\frac{x}{2} = \frac{\pi}{4} - \frac{\pi}{4} - \pi k$$

$$\frac{x}{2} = -\pi k$$

$$x = 2\pi k$$

333.

$$\sqrt{3} \sin x - \sin 2x = 0$$

$$\sqrt{3} \sin x - 2 \sin x \cos x = 0.$$

$$\sin x (\sqrt{3} - 2 \cos x) = 0$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \pm \arccos \frac{\sqrt{3}}{2} + 2\pi k$$

$$\sin x = 0$$

$$x = \pi k$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \pm \arccos \left(\frac{\sqrt{3}}{2} \right) + 2\pi k$$

$$x = \pm \frac{\pi}{6} + 2\pi k$$

$$x = \pi k$$

$$k \in \mathbb{Z}$$

$$x = \pm \frac{\pi}{6} + 2\pi k.$$

$$2\sin x + \operatorname{tg}(\pi - x) = 0$$

$$2\sin x - \operatorname{tg} x = 0$$

$$2\sin x - \frac{\sin x}{\cos x} = 0$$

$$\frac{2\sin x \cos x - \sin x}{\cos x} = 0$$

$$\frac{\sin x}{\cos x} \cdot (2\cos x - 1) = 0$$

$$\begin{cases} -\operatorname{tg} x = 0 \\ \cos x = \frac{1}{2} \end{cases} \begin{cases} x = \pi k \\ x = \pm \frac{\pi}{3} + 2\pi k \end{cases} \quad k \in \mathbb{Z}$$

9) $\operatorname{tg}^2 x - \operatorname{tg} x = 0$

$$\operatorname{tg} x (\operatorname{tg} x - 1) = 0$$

$$\begin{cases} -\operatorname{tg} x = 0 \\ \operatorname{tg} x = 1 \end{cases} \begin{cases} x = \pi k \\ x = \frac{\pi}{4} + \pi k \end{cases} \quad k \in \mathbb{Z}$$

336

12) $\cos 5x + \cos 9x = 0$

$$\frac{2 \cos 5x + 9x}{2} \cos \frac{9x - 5x}{2} = 0$$

$$2 \cos 7x \cos 2x = 0$$

$$\cos 7x = 0$$

$$\cos 2x = 0$$

$$7x = \pm \arccos 0 + 2\pi k$$

$$2x = \pm \arccos 0 + 2\pi k$$

$$7x = \pm \left(\frac{\pi}{2} + \pi k \right) + 2\pi k$$

$$2x = \pm \left(\frac{\pi}{2} + \pi k \right) + 2\pi k$$

$$x = \pm \frac{\pi(1+2k)}{14} + \frac{2\pi k}{7}$$

$$7x = \frac{\pi}{2} + \pi k$$

$$x = \frac{\pi}{14} + \frac{\pi k}{7}$$

$$2x = \frac{\pi}{2} + \pi k$$

$$x = \frac{\pi}{4} + \frac{\pi k}{2}$$

$$b) \sin 9x - \cos x = 0$$

$$\sin 9x - \sin \left(\frac{\pi}{2} - x \right) = 0$$

$$+ 2 \sin \frac{9x - \frac{\pi}{2} + x}{2} \cos \frac{9x + \frac{\pi}{2} - x}{2} = 0$$

$$+ 2 \sin \left(5x - \frac{\pi}{4} \right) \cos \left(4x + \frac{\pi}{4} \right) = 0$$

$$\sin \left(5x - \frac{\pi}{4} \right) = 0$$

$$5x - \frac{\pi}{4} = \pi k$$

$$\cos \left(4x + \frac{\pi}{4} \right) = 0$$

$$4x + \frac{\pi}{4} = \frac{\pi}{2} + \pi k$$

$$x = \left(\pi k - \frac{\pi}{4} \right) \cdot \frac{1}{5}$$

$$x = \frac{\pi k}{5} - \frac{\pi}{20}$$

$$x = \left(\frac{\pi}{2} - \frac{\pi}{4} + \pi k \right) \cdot \frac{1}{4}$$

$$x = \frac{\pi}{16} + \frac{\pi k}{4}$$

338-358.

337.

$$\text{w/ } \cos x = \sqrt{3} \sin x = \sqrt{2}$$

$$\cos x = \sqrt{3} \cos\left(\frac{x}{2} - x\right) = \sqrt{2}$$

344.

$$\text{w/ } \sin^4 \frac{x}{4} - \cos^4 \frac{x}{4} = \frac{1}{2}$$

$$\left(\sin^2 \frac{x}{4} - \cos^2 \frac{x}{4} \right) \left(\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} \right) = \frac{1}{2}$$

$$\cos \frac{x}{4} = \frac{1}{2}$$

$$- \cos \frac{x}{2} = \frac{1}{2}$$

$$\cos \frac{x}{2} = \frac{1}{2}$$

$$\frac{x}{2} = \pm \arccos\left(\frac{1}{2}\right) + 2\pi n$$

$$x = \pm \left(\arccos\left(\frac{1}{2}\right) + 2\pi n \right)$$

$$= \pm \arccos \frac{1}{2} + 2\pi n$$

$$= \pm \frac{2\pi}{3} + 2\pi n$$

$$x = \pm \frac{2\pi}{3} + 2\pi n$$

345/

$$\text{w/ } \sin^4 x + \cos^3 x = 1$$

$$\cos^3 x = 1 - \sin^4 x$$

$$\cos^3 x = (1 - \sin^2 x)(1 + \sin^2 x)$$

$$\cos^3 x = \cos^2 x (1 + \sin^2 x)$$

$$\cos^2 x (\cos x - 1 - \sin^2 x) = 0$$

$$\cos^2 x (\cos x - \cos^2 x) = 0$$

$$\cos^2 x = 0$$

$$\cos x = 0 \quad x = \frac{\pi}{2} + \pi k$$

$$\cos x (1 - \cos x) = 0$$

$$1 - \cos x = 0$$

$$\cos x = 1$$

$$\begin{cases} x = \frac{\pi}{2} + \pi k \\ x = 2\pi k \end{cases}$$

$$\cos x - 1 - \sin^2 x = 0$$

$$\cos x - 1 + \cos^2 x - 1 = 0$$

$$\cos x + \cos^2 x - 2 = 0$$

$$\cos^2 x + \cos x - 2 = 0$$

$$a = 1 + 8 = 3^2$$

$$\cos x = \frac{-1 \pm 3}{2} \quad \checkmark -2$$

$$\cos x = 1$$

$$x = 2\pi k$$

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$$\text{w/ } \sin^4 x + \cos^3 x = 1$$

$$\cos^3 x = 1 - \sin^4 x$$

$$\cos^3 x = (1 - \sin^2 x) (1 + \sin^2 x)$$

$$\cos^3 x = \cos^2 x (1 + \sin^2 x)$$

$$\cos^2 x (\cos x - 1 - \sin^2 x) = 0$$

$$\cos^2 x (\cos x - \cos^2 x) = 0$$

$$\cos^2 x = 0$$

$$\cos x = 0 \quad x = \frac{\pi}{2} + \pi k$$

$$\cos x (1 - \cos x) = 0$$

$$1 - \cos x = 0$$

$$\cos x = 1$$

$$\cos x - 1 - \sin^2 x = 0$$

$$\cos x - 1 + \cos^2 x - 1 = 0$$

$$\cos x + \cos^2 x - 2 = 0$$

$$\cos^2 x + \cos x - 2 = 0$$

$$a = 1, b = 1, c = -2$$

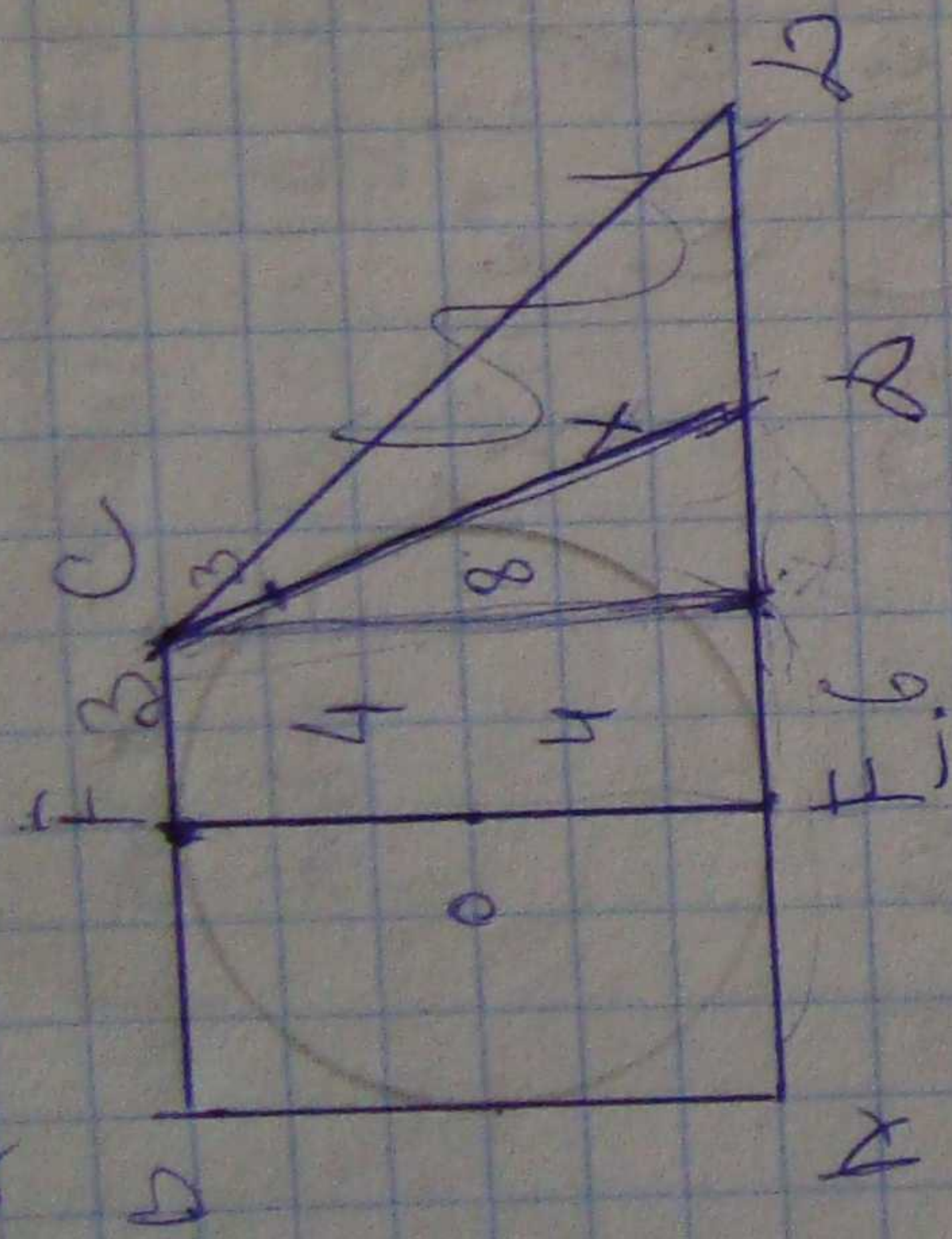
$$\cos x = \frac{-1 \pm 3}{2}$$

$$\cos x = 1$$

$$x = 2\pi k$$

$$\begin{cases} x = \frac{\pi}{2} + \pi k \\ x = 2\pi k \end{cases}$$

2.



$$BC = 6.$$

$$x = 4.$$

$$S = ?$$

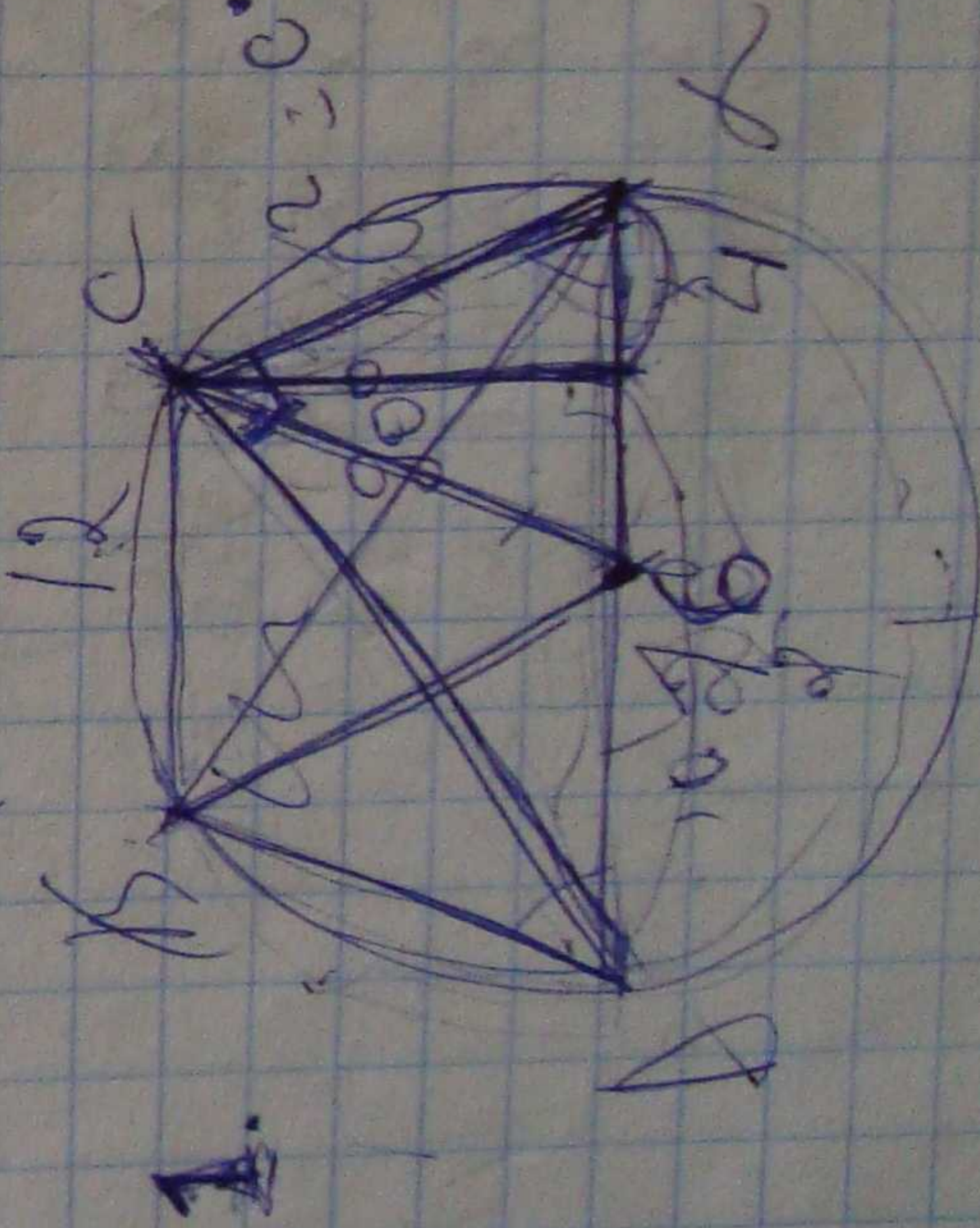
$$64 + (x-6)^2 = (3+x)^2$$

$$64 + x^2 + 36 - 12x = 9 + x^2 + 6x$$

$$100 - 91 - 18x = 0$$

$$x = \frac{91}{18}$$

$$1 \left(3 + \frac{91}{18} + 6 \right) \cdot 8 = 2$$



$$x = 8$$

$$AC = \sqrt{64 + 16} = 10.$$

2.

2



$z = y$
 $AD = z$
 $EF = AF$

$$2x \cdot x = 16 \cdot 2$$

$$x = 4$$

8.

Q. 1. $z = 4$

$$CB = 14.$$



2R - ?

$$7(4-x) = z \cdot y^2$$

$$20x = y^2$$

$$7(4-x) = 20x$$

$$28 = 20x$$

$$x = \frac{28}{20}$$

$$z = \sqrt{\frac{20 \cdot 28}{27}} = \sqrt{\frac{22 \cdot 15 \cdot 25}{27}}$$

$$z = \frac{2}{3} \sqrt{\frac{35}{3}}$$

$$\frac{5}{3} + 2\pi k$$

2/4/2020

$$p) \cos^4 x - \sin^3 x = 1$$

$$\sin^3 x = -(1 - \cos^4 x)$$

$$\sin^3 x = -(1 - \cos^2 x)(1 + \cos^2 x)$$

$$\sin^3 x = -\sin^2 x (1 + \cos^2 x)$$

$$\sin^3 x + \sin^2 x (1 + \cos^2 x) = 0$$

$$\sin^2 x (\sin x + 1 + \sin^2 x + 1) = 0$$

$$\sin^2 x = 0$$

$$\sin^2 x = 0$$

$$q) \sin 5x \cos 3x = \sin 9x \cos 8x$$

$$\sin \frac{1}{2} (\sin 8x + \sin 2x) = \frac{1}{2} (\sin 16x + \sin 2x)$$

$$\sin 8x \cos x = \frac{\sin 8x}{2}$$

$$\sin 8x (\cos x -$$

$$= \sin 16x - \sin 8x = 0$$

$$\sin 8x (2 \cos 2x - 1) = 0$$

$$\sin 8x = 0$$

$$8x = 0$$

$$2 \sin 4x \cos 12x = 0$$

$$\begin{cases} \sin 4x = 0 & \Rightarrow 4x = (-1)^n \pi \\ \cos 12x = 0 & \Rightarrow 12x = \frac{\pi}{2} + n\pi \end{cases}$$

2/4/2018

$$p) \cos^4 x - \sin^3 x = 1$$

$$\sin^3 x = 1 - \cos^4 x$$

$$\sin^3 x = (1 - \cos^2 x)(1 + \cos^2 x)$$

$$\sin^3 x = \sin^2 x (1 + \cos^2 x)$$

$$\sin^3 x + \sin^2 x (1 + \cos^2 x) = 0$$

$$\sin^2 x (\sin x + 1 - \sin^2 x + 1) = 0$$

$$\sin^2 x = 0$$

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$$q) \sin 5x \cos 3x = \sin 9x \cos 8x$$

$$\sin \frac{1}{2} (\sin 8x + \sin 2x) = \frac{1}{2} (\sin 6x + \sin 2x)$$

$$\sin 8x \cos x = \frac{\sin 8x}{2} = 0$$

$$\sin 8x (\cos x -$$

$$= \sin 6x - \sin 8x = 0$$

$$\sin 8x (2 \cos 2x - 1) = 0$$

$$\sin 8x = 0$$

$$8x = 0$$

$$2 \sin 4x \cos 12x = 0$$

$$\sin 4x = 0 \Rightarrow 4x = (-1)^n \pi$$

$$\cos 12x = 0 \Rightarrow 12x = \frac{\pi}{2} + n\pi$$

$$\text{Quadr. I. } 7\cos^4 2x + \sin^4 2x = 3$$

$$7\cos^4 2x = (1 - \sin^2 2x)(1 + \sin^2 2x)$$

$$7\cos^4 2x = \cos^2 2x (1 + \sin^2 2x)$$

$$7\cos^4 2x = \cos^2 2x (1 + \sin^2 2x) = 0.$$

$$\cos^2 2x (7\cos^2 2x - 1 - \sin^2 2x) = 0$$

$$\cos^2 2x (7\cos^2 2x - 1 - 1 + \cos^2 2x) = 0$$

$$\cos^2 2x = 0.$$

$$\cos 2x = 0.$$

$$2x = \frac{\pi}{2} + \pi k$$

$$x = \frac{\pi}{4} + \frac{\pi k}{2}$$

$$8\cos^2 2x - 2 = 0.$$

$$8\cos^2 2x = 2$$

$$\cos^2 2x = \frac{1}{4}$$

$$\cos 2x = \pm \frac{1}{2} \quad \cos 2x = -\frac{1}{2}$$

$$2x = \pm \arccos \frac{1}{2} + 2\pi k = \pm \frac{\pi}{3} + 2\pi k$$

$$x = \pm \frac{\pi}{6} + \pi k$$

$$x \cos 2x = \arccos\left(-\frac{1}{2}\right) + 2\pi k =$$

$$= \arccos\left(\pm\left(\pi - \frac{\pi}{3}\right) + 2\pi k\right) = \pm \frac{2\pi}{3} + 2\pi k$$

$$x = \pm \frac{\pi}{3} + \pi k$$



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